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*Matrix Darboux chain in classical mechanics and spectral theory*

The study of the motion of a rigid body about a fixed point is one of the most classical problems of mechanics going back to Euler. Remarkably, only in 1980s Reyman and Bogoyavlenskij independently discovered that the corresponding system turns out to be integrable in the Newtonian field with quadratic potential.

Following our recent joint work with V.E. Adler, I will show that the Brun-Bogoyavlenskij system is a special reduction of the period-one closure of the Darboux chain for the matrix Schrödinger operators. This will lead to the interesting new examples of such operators (including some finite-gap matrix versions of the Mathieu operator) with explicitly described spectrum.

I will finish with a brief discussion of some open problems in integrable systems and related areas.