How to engineer Celestial conformal field theories?

Shamik Banerjee
NISER, Bhubaneswar, India
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Shamik Banerjee, Nishant Gupta, Sagnik Mishra

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What is Celestial CFT?

- Celestial CFT is usually defined as a CFT whose correlation functions are given by the (Mellin transform) scattering amplitudes in asymptotically flat space time.
- This is a useful definition and has taught us many new things especially about symmetries of scattering amplitudes.
- However, to make progress we have to first understand CCFT as field theory in its own right without any reference to scattering amplitudes. (See for example the talk by Ana.)
- So for the purpose of this talk a Celestial CFT_d is any QFT_d with $\mathrm{ISO}(d+1,1)$ symmetry and on which $\mathrm{SO}(d+1,1)$ acts as the conformal group.

An analogy

- We do not define CFTs as QFTs which compute string scattering amplitude in AdS.
- In fact, most of the breakthroughs in our understanding of CFTs were in the pre AdS-CFT era.
- Thankfully someone studying percolation using CFT does not have to learn String Theory on AdS!

What we do

- In this talk I discuss a method to "engineer" Celestial CFT_d using AdS_{d+1}-CFT_d duality.
- We use gravity/string theory. So this is still not intrinsic.
- However, we can use powerful tools of AdS_3-CFT_2 to study Celestial CFT_2.
- This also shows that Celestial CFTs may appear in places where there is no flat space (holography). This is encouraging and expected!

Near-Boundary limit in Euclidean AdS_3

- Let us consider $EAdS_3$ in Poincare coordinates. The line element is given by

$$ds^2 = \frac{d\eta^2 + dz\, d\bar{z}}{\eta^2} \tag{1}$$

- The *conformal* Killing vector fields (CKVs) of AdS_3 can be written in the following form

$$\begin{aligned}
\mathcal{L}_m &= -z^{m+1}\,\partial_z + \frac{1}{2}\,m(m+1)\,\eta^2\,\partial_{\bar{z}} - \frac{1}{2}(m+1)\,z^m\,\eta\,\partial_\eta \\
\bar{\mathcal{L}}_m &= -\bar{z}^{m+1}\,\partial_{\bar{z}} + \frac{1}{2}\,m(m+1)\,\eta^2\,\partial_z - \frac{1}{2}(m+1)\,\bar{z}^m\,\eta\,\partial_\eta \\
\mathcal{P}_{r,s} &= -z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\partial_\eta + 2\eta\left(s+\frac{1}{2}\right)z^{r+s}\partial_z + 2\eta\left(r+\frac{1}{2}\right)\bar{z}^{r+s}\partial_{\bar{z}} \\
&\quad + (r+\frac{1}{2})(s+\frac{1}{2})\,\eta^2\,\partial_\eta
\end{aligned} \tag{2}$$

where $m \in \{0, \pm 1\}$ and $(r,s) \in \{\pm\frac{1}{2}\}$.

- These vector field obey commutation relations,

$$
\begin{aligned}
&[\mathcal{L}_m, \mathcal{L}_n] = (m-n)\mathcal{L}_{m+n}\,, \quad \left[\bar{\mathcal{L}}_m, \bar{\mathcal{L}}_n\right] = (m-n)\bar{\mathcal{L}}_{m+n} \\
&[\mathcal{L}_m, \mathcal{P}_{r,s}] = \frac{1}{2}\left(m-2r\right)\mathcal{P}_{m+r,s}\,, \quad \left[\bar{\mathcal{L}}_m, \mathcal{P}_{r,s}\right] = \frac{1}{2}\left(m-2s\right)\mathcal{P}_{r,m+s} \\
&[\mathcal{P}_{r,s}, \mathcal{P}_{r',s'}] = 2\left(\epsilon_{rr'}\bar{\mathcal{L}}_{s+s'} + \epsilon_{ss'}\mathcal{L}_{r+r'}\right)
\end{aligned} \tag{3}
$$

where $\epsilon_{-\frac{1}{2}\frac{1}{2}} = -\epsilon_{\frac{1}{2}-\frac{1}{2}} = 1$.

- The commutators is the Lie algebra of $SO(4,1)$ which is the group of conformal transformations of EAdS_3

- To zoom in near the boundary we define the new coorrdinate $\tilde{\eta}$ by

$$\eta = \epsilon\tilde{\eta}, \;\; \epsilon \to 0 \tag{4}$$

and take $\epsilon \to 0$ at fixed $\tilde{\eta}$.

- In terms of $\tilde{\eta}$ the CKVs become

$$\begin{aligned}
\mathcal{L}_m &= -z^{m+1}\,\partial_z - \frac{\tilde{\eta}}{2}(m+1)\,z^m\,\partial_{\tilde{\eta}} + \frac{\epsilon^2\tilde{\eta}^2}{2}\,m(m+1)\,\partial_{\bar{z}} \\
\bar{\mathcal{L}}_m &= -\bar{z}^{m+1}\,\partial_{\bar{z}} - \frac{\tilde{\eta}}{2}(m+1)\,\bar{z}^m\,\partial_{\tilde{\eta}} + \frac{\epsilon^2\tilde{\eta}^2}{2}\,m(m+1)\,\partial_z \\
\mathcal{P}_{r,s} &= -z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\frac{\partial_{\tilde{\eta}}}{\epsilon} + \epsilon\left[2\,\tilde{\eta}\left(s+\frac{1}{2}\right)\,z^{r+s}\partial_z + 2\,\tilde{\eta}\left(r+\frac{1}{2}\right)\,\bar{z}^{r+s}\partial_{\bar{z}}\right. \\
&\quad \left. + (r+\frac{1}{2})(s+\frac{1}{2})\,\tilde{\eta}^2\,\partial_{\tilde{\eta}}\right].
\end{aligned} \tag{5}$$

- Now we take $\epsilon \to 0$ keeping $\tilde{\eta}$ fixed and define the rescaled vector fields $P_{r,s}$

$$P_{r,s} = \epsilon\,\mathcal{P}_{r,s}, \;\; \epsilon \to 0 \tag{6}$$

- In the limit $\epsilon \to 0$ we obtain the following vector fields,

$$\begin{aligned} L_m &= -z^{m+1}\,\partial_z - \frac{1}{2}(m+1)\,z^m\,\tilde{\eta}\,\partial_{\tilde{\eta}}, \\ \bar{L}_m &= -\bar{z}^{m+1}\,\partial_{\bar{z}} - \frac{1}{2}(m+1)\,\bar{z}^m\,\tilde{\eta}\,\partial_{\tilde{\eta}}, \\ P_{r,s} &= -z^{r+\frac{1}{2}}\,\bar{z}^{s+\frac{1}{2}}\,\partial_{\tilde{\eta}}, \end{aligned} \tag{7}$$

- The vector fields satisfy the ISO(3,1) algebra,

$$\begin{gathered} [L_m, L_n] = (m-n)L_{m+n}\,, \qquad \left[\bar{L}_m, \bar{L}_n\right] = (m-n)\bar{L}_{m+n} \\ [L_m, P_{r,s}] = \frac{1}{2}\,(m-2r)\;P_{m+r,s}\,, \quad \left[\bar{L}_m, P_{r,s}\right] = \frac{1}{2}\,(m-2s)\;P_{r,m+s} \\ [P_{r,s}, P_{r',s'}] = 0. \end{gathered} \tag{8}$$

- So when we zoom in near the boundary the conformal isometry group SO(4,1) contracts to the Poincare group ISO(3,1).

- Note that the subgroup $SO(3,1) \subset ISO(3,1)$ acts on the two dimensional boundary as conformal transformations.

- The four translations $\subset ISO(3,1)$ cannot be realized geometrically but are internal symmetries.

An almost self-evident statement

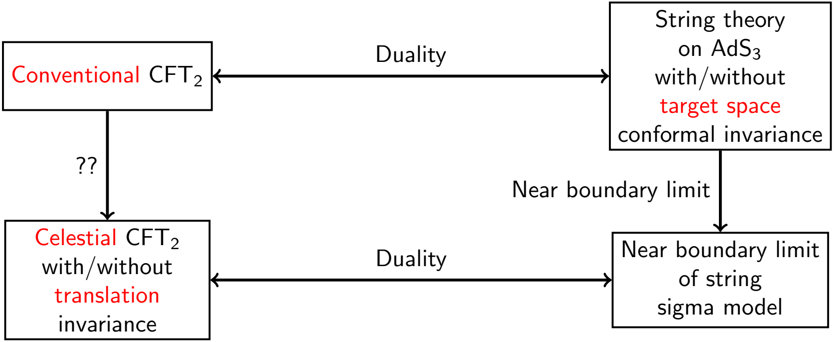

Figure: Diagrammatic representation of our proposal

- After we take the near-boundary limit in the bulk the dual boundary theory should have (I)SO(3, 1) symmetry with the SO(3, 1) acting as the conformal group.

Near boundary limit of bosonic string theory on AdS_3 with NS-NS B field

- Has been studied for a long time and lot of recent works on this in the string theory community.

- Conventional string theories do not have target space conformal invariance and in the low energy limit give Einstein gravity rather than conformal gravity. Therefore in the near boundary limit string theories on AdS should be dual to Celestial CFTs with only conformal invariance but not Poincare invariance.

- It will be very interesting to study similar limit for Twistor String Theory with target space conformal invariance.

- We write the the AdS_3 metric as

$$ds^2 = l^2 \left(d\phi^2 + e^{2\phi} d\gamma d\bar{\gamma} \right) \tag{9}$$

where l is the AdS_3 radius.

- Bosonic part of the worldsheet Lagrangian on AdS_3 is given by the Wess-Zumino-Witten model for the coset $SL(2,\mathbb{C})/SU(2)$ and can be written as

$$S = \frac{2l^2}{l_s^2} \int d^2w \left(\partial\phi\bar{\partial}\phi + e^{2\phi}\bar{\partial}\gamma\partial\bar{\gamma} \right) + S_{\text{int}} \tag{10}$$

where S_{int} is the action describing string propagation on compact internal space.
(Giveon-Kutasov-Seiberg, de Boer-Ooguri-Robins-Tannenhauser, Maldacena-Ooguri)

- Now to take the near boundary limit it is useful to write the action in the first order form as

$$S = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+}\sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} - \beta\bar{\beta}e^{-\frac{2}{\alpha_+}\phi} \right) + S_{\text{int}} \tag{11}$$

where (β, γ) are bosonic holomorphic fields with weights $(1, 0)$ and $(0, 0)$. $(\bar{\beta}, \bar{\gamma})$ are the corresponding antiholomorphic fields. The parameter α_+ is related to the string length l_s and the AdS radius l by

$$\alpha_+^2 = 2k - 4 = 2\frac{l^2}{l_s^2} - 4 \tag{12}$$

R is the worldsheet curvature.

- In the near boundary limit $\phi \to \infty$ the last term in the action (11) can be neglected and we get the free worldsheet action,

$$S_{NB} = \int d^2w \left(\partial\phi\bar{\partial}\phi - \frac{2}{\alpha_+}\sqrt{g}R\phi + \beta\bar{\partial}\gamma + \bar{\beta}\partial\bar{\gamma} \right) + S_{\text{int}} \tag{13}$$

(Giveon-Kutasov-Seiberg, de Boer-Ooguri-Robins-Tannenhauser)

Some properties of S_{NB}

- The equation of motion of $\beta(\bar{\beta})$

$$\bar{\partial}\gamma = 0, \ \partial\bar{\gamma} = 0$$

require γ $(\bar{\gamma})$ to be a holomorphic (anti-holomorphic) map from the worldsheet to the boundary sphere (S^2) in the target space. These wrapped strings are also known as the "Long-Strings".

- The dual or space-time CFT_2 describing the long-strings has central charge $c = 6kp$ where p is the number of times the worldsheet wraps the boundary sphere. Seiberg and Witten argued that the space-time CFT_2 which describes the long strings has a Liouville sector with background charge (for $p = 1$)

$$Q = (k-3)\sqrt{\frac{2}{k-2}} \qquad (14)$$

and central charge

$$c_L = 1 + 3Q^2 = 1 + 6\frac{(k-3)^2}{k-2} \qquad (15)$$

- The affine symmetries of the free action S_{NB} can be "lifted" to the space-time and become affine symmetries of the dual CFT_2.

- To be more precise, if the worldsheet theory has an affine $\hat{G}$ symmetry with level k' then after lifting to the space time it becomes an affine $\hat{G}$ symmetry of the dual CFT_2 with level $\tilde{k}$ given by $\tilde{k} = pk'$. Here p is the number of times the string worldsheet wraps the boundary sphere.
(Giveon-Kutasov-Seiberg)

- Similarly the affine $SL_2 \times SL_2$ symmetry of S_{NB}, after lifting to the space time, becomes the Virasoro algebra of the dual CFT_2.
(Giveon-Kutasov-Seiberg)

Long string CFT = Celestial CFT

- The space-time CFT_2 dual to the string theory described by the worldsheet action S_{NB} is an example of a Celestial CFT_2 (with only Lorentz invariance). The Celestial CFT_2 contains a Liouville sector which describes the radial fluctuation of the long string worldsheet.

- Other fields which live on the long string worldsheet are the conformal fields which describe the position of the string in the compact space.

- Moreover, the infinite dimensional symmetries of the Celestial CFT_2 can be thought of as lifts of the infinite dimensional symmetries on the worldsheet.

Comparison to literature

- In very interesting recent works examples of Celestial CFT_2 were constructed which are Liouville theory coupled to various matter fields. These theories are (Lorentz) conformally invariant but not Poincare invariant. These theories compute the tree level MHV gluon scattering amplitudes.
(Steiberger-Taylor-Zhu, Melton-Sharma-Strominger-Wang, Donnay-Giribet-Valsesia)

- Our construction in no way explains their construction.

- However, the existence of the Liouville sector in both examples of celestial CFT_2 suggests that Liouville sector could be a feature of a large class of Celestial CFT_2s.

Future Directions

- Introduce RR-flux.

- Study the near boundary limit of (Twistor) string theory with target space conformal invariance and the nature of the dual Celestial CFT_2 with $\mathrm{ISO}(3,1)$ invariance.

- What does near boundary limit correspond to in the field theory side? In other words, our construction suggests that

$$\text{Conventional } \mathrm{CFT}_2 \rightarrow \text{Celestial } \mathrm{CFT}_2$$

Find out an explicit realization of this which does not use AdS-CFT.