What is boundary operator?

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Boundary Operators in flat space-time by Extrapolation

- One way to address this problem is to start from the Inverse LSZ formula for massless particles

$$G(\{x_i, \delta_i\}) = \prod_i \int d\mu_{p_i} e^{-i\delta_i p_i \cdot x_i} S(\{\delta_i p_i\}), \ \delta_i = \pm 1, \ x_i \to \infty \qquad (1)$$

where $d\mu_{p_i} = \frac{d^{D-1}p_i}{(2\pi)^{D-2} 2\omega_{p_i}}$ is the Lorentz invariant integration measure on the cone and $p^0 = \omega_p = |\vec{p}|$. (Minwalla and collaborators, 2311.03443[hep-th])

- G is the bulk time-ordered Green's function and S is the S-matrix.

Celestial Amplitudes

- Massless Celestial amplitudes are defined as (Pasterski-Strominger-Shao)

$$\mathcal{A}(\{z_i, \bar{z}_i, \Delta_i, \delta_i\}) = \prod_{i=1}^{n} \int_0^\infty d\omega_i \omega_i^{\Delta_i - 1} S(\{\delta_i \omega_i, z_i, \bar{z}_i\}) \qquad (2)$$

where massless momentum is parametrized as

$$p^\mu = \omega q^\mu(z, \bar{z}) = \omega \left(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z}\right), \ \omega \geq 0 \qquad (3)$$

- This suggests that at least the (naive) Celestial boundary operators are

$$a\left(z, \bar{z}, \Delta, \delta\right) = \int_0^\infty d\omega \omega^{\Delta - 1} a(\delta\omega, z, \bar{z}) \qquad (4)$$

where $a(\delta\omega, z, \bar{z})$ is the (annihilation) creation operator.

Boundary Operators from inverse LSZ

- One can play with the Inverse LSZ and get

$$
\begin{aligned}
&G(\{x_i \to \infty, \delta_i\}) \\
&= \prod_i \frac{1}{(2\pi)^2} \int \frac{d\Delta_i}{2\pi} \Gamma(\Delta_i) \frac{(-i\delta_i)^{\Delta_i}}{R_i^{\Delta_i}} \int d^2 w_i \frac{1}{|z_i - w_i|^{2\Delta_i}} \mathcal{A}(\{w_i, \bar{w}_i, 2-\Delta_i, \delta_i\}) \\
&+\text{Contact Terms} \\
&= \prod_i \frac{1}{4\pi} \int \frac{d\Delta_i}{2\pi} \Gamma(1-\Delta_i) \frac{(-i\delta_i)^{\Delta_i}}{R_i^{\Delta_i}} \tilde{\mathcal{A}}(\{z_i, \bar{z}_i, \Delta_i, \delta_i\}) \\
&+\text{Contact Terms}, \\
&R_i \to \infty
\end{aligned}
\tag{5}
$$

(Banerjee, 2406.06690 [hep-th], JHEP)

- $\tilde{\mathcal{A}}(\{z_i, \bar{z}_i, \Delta_i, \delta_i\})$ is the Shadow Celestial Amplitude.

- Schematically

$$G(\{x_i \to \infty, \delta_i\}) \sim \sum_{\{\Delta_i\}} \prod_i R_i^{-\Delta_i} \tilde{\mathcal{A}}(\{z_i, \bar{z}_i, \Delta_i, \delta_i\}) \tag{6}$$

- So the boundary correlation function obtained from Inverse LSZ is

$$\tilde{\mathcal{A}}(\{z_i, \bar{z}_i, \Delta_i, \delta_i\}) = \prod_i \int d^2 w_i \frac{1}{|z_i - w_i|^{2\Delta_i}} \mathcal{A}(\{w_i, \bar{w}_i, 2 - \Delta_i, \delta_i\}) \tag{7}$$

- So, Boundary operator according to Inverse LSZ is

$$\tilde{a}(z, \bar{z}, \Delta, \delta) = \int d^2 w \frac{1}{|z - w|^{2\Delta}} a(w, \bar{w}, 2 - \Delta, \delta) \tag{8}$$

Good News!

- Shadow Celestial amplitudes are non-distributional.

- Stress tensor itself is the shadow of the subleading soft graviton.

- $w_{1+\infty}$ currents are also light transform (half-shadow) of the soft graviton operators.

- One is tempted to say that:
  Shadow Celestial CFT_2 is the true boundary theory.

- So the boundary theory is an ordinary CFT_2 with non-distributional correlation functions.

- More evidence from AdS_3-CFT_2 to follow.