

Vertex operator algebras, higher genus partition functions and Teichmüller modular forms

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Classical modular forms are holomorphic functions on the upper half plane H satisfying certain functional equations related to the action of the modular group $SL(2, \mathbb{Z})$. They are deeply related to the geometry of the moduli space of genus one compact complex curves. Vertex operator algebras (VOAs) give a mathematical description of chiral two-dimensional conformal field theories (chiral CFTs). VOAs with trivial representation theory are called holomorphic and their genus 1 partition functions are known to give rise to classical modular forms. Teichmüller modular forms are higher genus generalizations of classical modular forms. In this talk I will review some recent results of an ongoing joint work with Giulio Codogni. If V is a holomorphic VOA with central charge c we show that, for every non-negative integer g , the genus g partition function of V gives rise in a natural way to a Teichmüller modular form of weight $c/2$. This gives strong constraints on the partition functions of holomorphic VOAs. Moreover, we clarify the relation between unitary VOAs having the same genus g partition function for all g . In various cases this relation can be also related to certain subfactors arising from inclusions of VOAs through the correspondence with conformal nets. Finally, we relate the important open problem of the uniqueness of the moonshine VOA with a weak form of the Harrison-Morrison slope conjecture about the geometry of the moduli spaces of compact Riemann surfaces.