The harmonic oscillator on the Moyal-Groenewold plane

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Let $A = -\Delta + |x|^2$ be the harmonic oscillator which generates a submarkovian semigroup on $L^p(R^d)$. It is well-known that its spectral multipliers $m(A)$ are bounded on L^p, $1 < p < \infty$, provided that $m : (0, \infty) \to C$ is $\lfloor \frac{d}{2} \rfloor + 1$ times differentiable with a certain L^2 type control on its derivatives (Hörmander functional calculus).

On the other hand, A can be written as a square sum

$$A = \sum_{k=1}^{d} (i\partial_k)^2 + \sum_{l=1}^{d} x_l^2$$

where the operators $iA_k = -\partial_k$ and $iB_l = ix_l$ generate bounded (translation and modulation) c_0-groups on L^p, that obey simple, so-called canonical, commutation relations (CCR).

Recently, van Neerven, Portal and Sharma showed in a series of papers via a transference technique and square function estimates, that square sum operators (as A above) for quite general bounded c_0-groups with CCR inherit the Hörmander functional calculus from A.

We show that this also holds for c_0-groups acting on noncommutative L^p-spaces. Along the way of proof, an important step is a new method to generate square function estimates on noncommutative L^p.

An application are L^p bounded Hörmander spectral multipliers of the harmonic oscillator on the Moyal plane (also called quantum euclidean space), which are particular noncommutative pseudo-differential operators studied by González-Pérez, Junge and Parcet.

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