The onset of filamentation on vorticity interfaces

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Vorticity measures the local spin in a fluid. In a planar (two-dimensional) fluid, vorticity is a scalar field, and under ideal conditions (no viscosity and no density variations), it is conserved following every fluid "particle" as it moves under the action of the velocity field. That field is determined from the global distribution of vorticity, and as such the fluid motion is determined by non-local interactions. Moreover, the transport of vorticity is fundamentally non-linear, making even "ideal" fluid motion mathematically challenging. One of the simplest situations one may consider is a vorticity interface, a curve separating two uniform regions of vorticity. In this case, the evolution of the curve depends only on its instantaneous shape, yet it is still non-local and non-linear. In 1880, Lord Kelvin (William Thompson) studied the behaviour of small disturbances to a circular patch of uniform vorticity. A circular patch induces a steady, rotating flow. Kelvin found that the disturbances simply oscillate, with a frequency equal to half of the jump in vorticity across the interface - independent of the wavelength of the disturbance. He conjectured that the non-linear dynamics would exhibit a progressive steepening of the disturbance, ultimately culminating in filaments being extruded from the interface. This remarkable insight was confirmed by the present author more than a century later in 1988. This talk will examine new results concerning the weakly non-linear dynamics of vorticity interfaces. In particular, a cubically non-linear equation of universal form is shown to characterise the steepening of any disturbance, ultimately leading to "filamentation", the apparently endless extrusion of filaments every wave period.