Carrollian perspective on the flat limit of AdS/CFT

AdS/CFT meets Carrollian & Celestial Holography

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- Introduction and Motivations
- Elements of Celestial & Carrollian holography
- Flat limit of AdS/CFT
- Conclusions

Introductions and Motivations

Introduction and Motivations

- Holography of $3+1$ dimensional bulk flat spacetime has seen an avalanche of work following up on the discovery of the IR triangle (Strominger [2013]; He, Lysov, Mitra, Strominger [2014]; Strominger [2014]; Pasterski [2015])

- Earlier work that predated this was done mostly in $2+1$ dimensional bulk (Bagchi [2010]; Bagchi, Fareghbal [2012]; Bagchi, Detournay, Fareghbal, Simón [2012]; Barnich [2012])

- Given the robustness of AdS/CFT, one would hope a suitable large AdS radius limit, or equivalently a large N limit of the dual CFT, would uncover features of flat space holography.

- Historically, the large AdS radius limit has been used to extract flat space S-matrices: Polchinski [1999] and Susskind [1999] approached the problem from $N \to \infty$ of the CFT

Introduction and Motivations

- Later work (Balasubramaniam, Giddings, Lawrence [1999]; Giddings [1999]; Gary, Giddings, Penedones [2009]) focussed on obtaining S-matrices in the bulk of AdS by carefully constructing scattering states (suitably defined) in AdS.

- Finally, all these works were generalized by Penedones [2010]: AdS Scattering amplitude was given by the Mellin space representation (Mack [2009]) of the CFT correlation functions. A suitable rescaling of this amplitude in the large AdS radius limit encoded the bulk S-matrix.

- These works don't consider the dual boundary description of the emergent Poincaré symmetry in the bulk.

- Our work (Bagchi, PD, Dutta [2303.07388, 2311.11246]) attempts to address this question by considering AdS Witten diagrams following up on work done by de Gioia and Raclariu [2022].

Introduction and Motivations

- The asymptotic symmetries of $3+1$ dimensional flat space are BMS_4 (Bondi, Van der Burg, Metzner [1961] and Sachs [1962])

$$[L_n, L_m] = (n-m)\,L_{m+n}\,, \quad [\bar{L}_n, \bar{L}_m] = (n-m)\,\bar{L}_{m+n}\,, \quad [M_{r,s}, M_{p,s}] = 0\,,$$

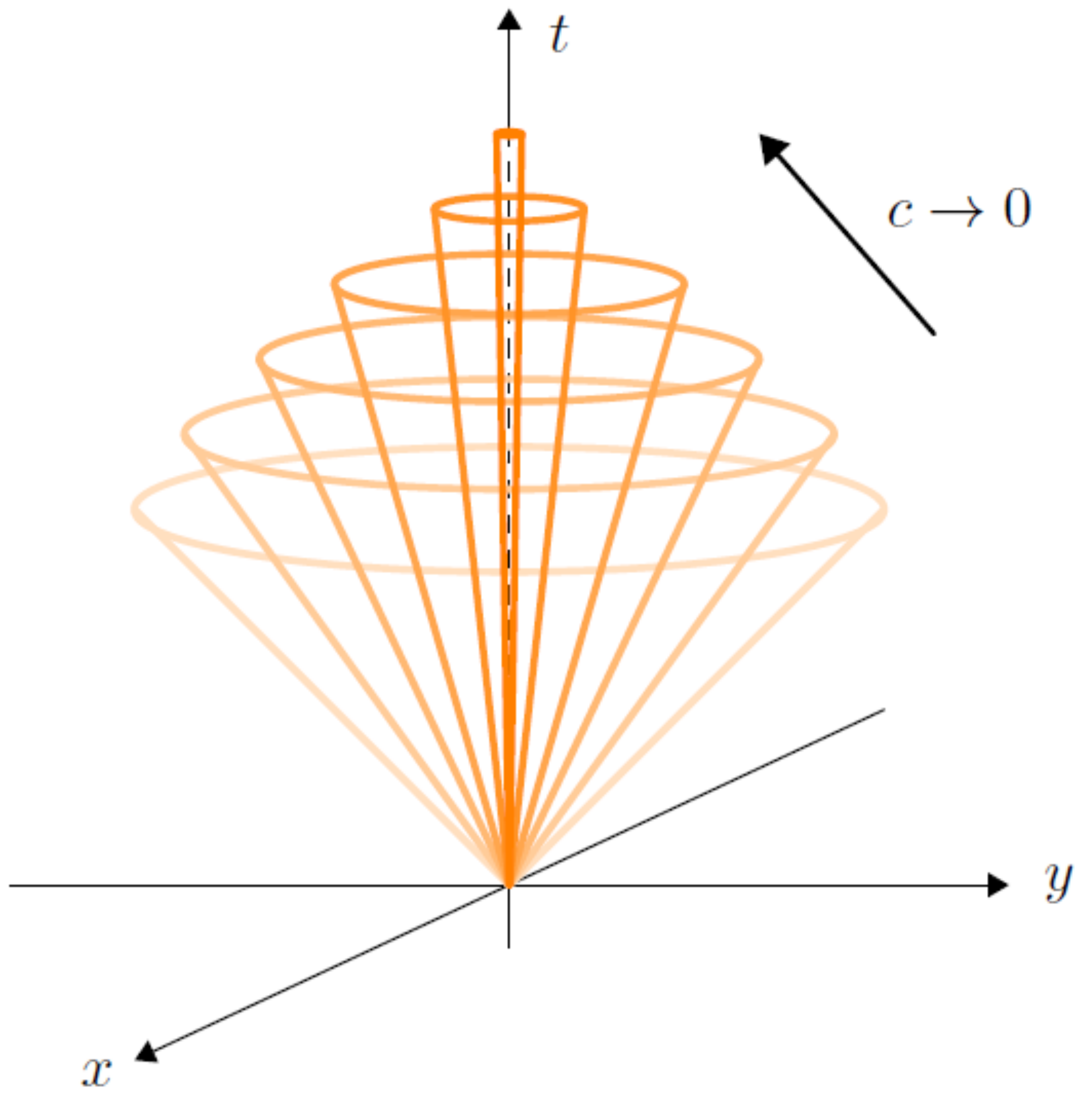
$$[L_n, M_{r,s}] = \left(\frac{n+1}{2} - r\right) M_{r+n,s}\,, \qquad [\bar{L}_n, M_{r,s}] = \left(\frac{n+1}{2} - s\right) M_{r,s+n}$$

$L_{0,\pm1}, \bar{L}_{0,\pm1}, M_{00}, M_{10}, M_{01}, M_{11}$ form the Poincaré algebra. This algebra is isomorphic to the infinite dimensional lift of the conformal Carrollian algebra. A candidate dual to asymptotically flat holography.

Recent refs: Donnay, Fiorucci, Herfray, Ruzziconi [2022,2022]; Bagchi, Banerjee, Basu, Dutta [2022],…

Introduction and Motivations

- Carroll algebra: Contraction of the Poincaré algebra ($c \to 0$)



$$x_i \to x_i\,, \qquad t \to \epsilon t\,, \qquad \epsilon \to 0$$

PC: Bagchi, Kolekar, Shukla [2023]

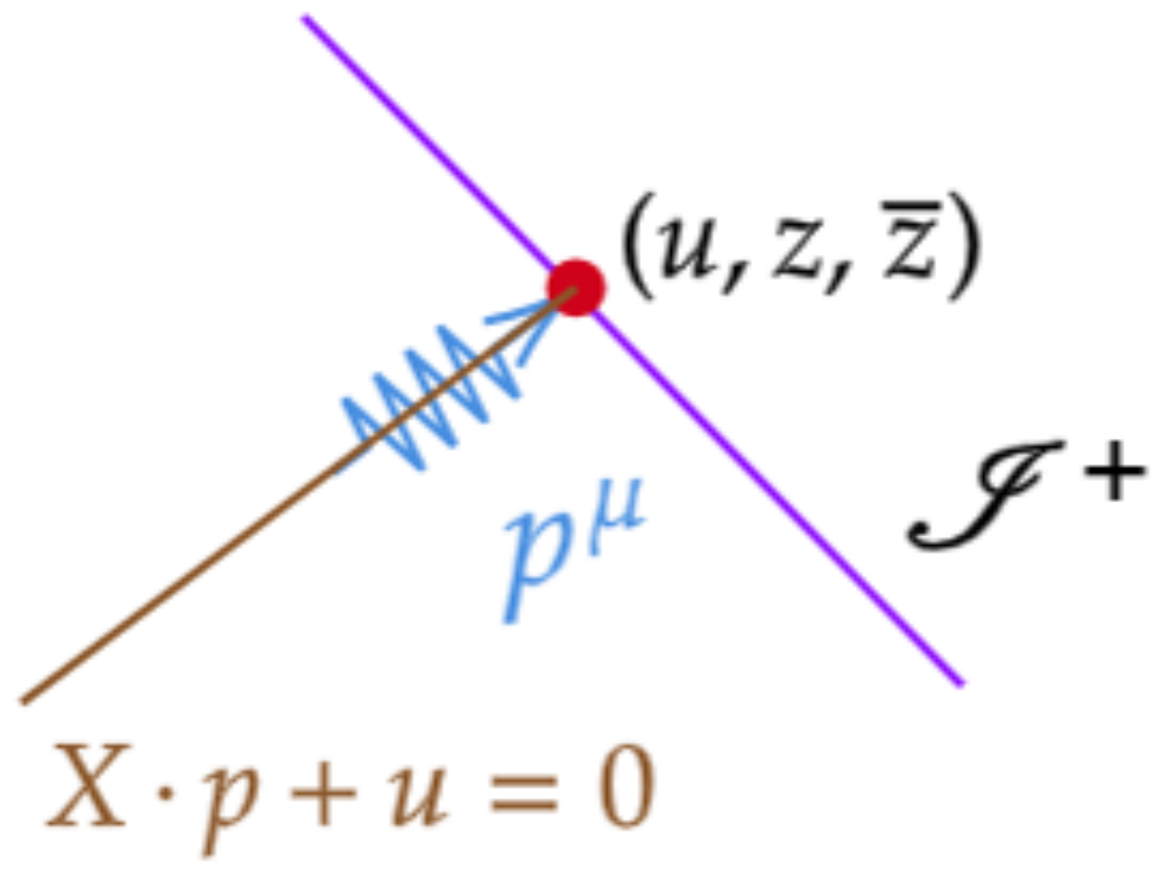
One can do a conformal extension

Carroll algebra is also an isometry algebra of $$ds^2 = \lim_{c\to 0} - c^2 dt^2 + d\vec{x}^2 = 0.dt^2 + d\vec{x}^2$$

Elements of Celestial & Carrollian Holography

How symmetries act on the null boundary

- For a massless particle reaching $\mathscr{I}^+$, the momenta p^μ can be parametrized by it's energy ω and the angle on S^2 $(z, \bar{z})$



$$p^\mu = \omega\,(1 + z\bar{z}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z})\,, \qquad z = \frac{p^1 + ip^2}{p^0 + p^3}$$

- Bulk Lorentz transformations Λ ~ Boundary SL$(2,\mathbb{C})$ transformations

$$p \to \Lambda p \implies z \to z' = \frac{p'^1 + ip'^2}{p'^0 + p'^3} = \frac{az + b}{cz + d}$$

Bulk translations $X^\mu \to X'^\mu = X^\mu + \ell^\mu$ ~ Shifts in u

$$u \to u' = u + (\ell^0 + \ell^3) - (\ell^1 - i\ell^2)\,z - (\ell^1 + i\ell^2)\,\bar{z} + (\ell^0 - \ell^3)\,z\bar{z}$$

Quantities covariant under the symmetries

- One particle states $|p^\mu, \sigma\rangle = a^\dagger_{p,\sigma}|0\rangle$ can be expressed as conformal primaries ($\epsilon = \pm 1$ -Creation/annihilation operators) with $\Delta = h + \bar{h}$ and $\sigma = h - \bar{h}$

$$\mathcal{O}^{\epsilon}_{h,\bar{h}}(z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1}\, a(\epsilon\omega, z, \bar{z}, \sigma)$$

$$\mathcal{O}'^{\epsilon}_{h,\bar{h}}(z,\bar{z}) = \frac{1}{(cz+d)^{2h}} \frac{1}{(\bar{c}\bar{z}+\bar{d})^{2\bar{h}}}\, \mathcal{O}^{\epsilon}_{h,\bar{h}}\left(\frac{az+b}{cz+d}, \frac{\bar{a}\bar{z}+\bar{b}}{\bar{c}\bar{z}+\bar{d}}\right)$$

- This establishes a Holographic correspondence through the Mellin transform (Pasterski, Shao, Strominger [2017]; Pasterski, Shao [2017])

$$\langle \mathcal{O}^{\epsilon_1}_{h_1,\bar{h}_1}(z_1,\bar{z}_1)\, \mathcal{O}^{\epsilon_2}_{h_2,\bar{h}_2}(z_2,\bar{z}_2)\ldots\rangle = \int \prod_{i=1}^{n} d\omega_i\, \omega_i^{\Delta_i-1}\, \mathcal{S}_n(\{\omega_i, z_i, \bar{z}_i, \sigma_i\})$$

Issues of covariance under translations

- Consider time translations generated by the Hamiltonian H

$$\delta_H \mathcal{O}^{\epsilon_i}_{h_i,\bar{h}_i}(z_i,\bar{z}_i) = \int_0^\infty d\omega_i\, \omega_i^{\Delta_i-1}\, [H, a(\epsilon_i\omega_i, z_i, \bar{z}_i, \sigma_i)]$$

$$= \int_0^\infty d\omega_i\, \omega_i^{\Delta_i-1}[-\epsilon_i\omega_i(1+z_i\bar{z}_i)]a(\epsilon_i\omega_i, z_i, \bar{z}_i, \sigma_i)$$

$$= -\epsilon_i(1+z_i\bar{z}_i)\mathcal{O}^{\epsilon_i}_{h_i+\frac{1}{2},\bar{h}_i+\frac{1}{2}}(z_i,\bar{z}_i)$$

Thus, even though the bulk Lorentz group acts as SL$(2,\mathbb{C})$, the action of translations shifts the conformal dimension!

- The tree level graviton amplitudes are UV divergent in the Mellin basis (Stieberger, Taylor [2018]; Puhm [2019]).

Remedy: Null time evolution

- Consider the time evolution of the operator $\mathcal{O}_{h,\bar{h}}(z,\bar{z})$ by Hamiltonian H

$$\Phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z}) = e^{-iHU}\mathcal{O}_{h,\bar{h}}(z,\bar{z})e^{iHU} = \int_0^{\infty} d\omega\, \omega^{\Delta-1}\, e^{-i\epsilon\omega u} a(\epsilon\omega, z, \bar{z})$$

Here $u = U(1+z\bar{z})$. This still transforms as a SL$(2,\mathbb{C})$ primary (Banerjee [2018]) but now with the advantage that it is covariant under translations

$$\Phi^{\epsilon}_{h,\bar{h}}(u,z,\bar{z}) \to \Phi'^{\epsilon}_{h,\bar{h}}(u',z',\bar{z}') = \Phi^{\epsilon}_{h,\bar{h}}(u+p+qz+r\bar{z}+sz\bar{z}, z, \bar{z})$$

One can propose a modified Mellin transform (Bagchi, Banerjee, Basu, Dutta [2022]) using these new "Carrollian primaries"

$$\langle \Phi^{\epsilon_1}_{h_1,\bar{h}_1}(u_1,z_1,\bar{z}_1)\, \Phi^{\epsilon_2}_{h_2,\bar{h}_2}(u_2,z_2,\bar{z}_2)\ldots\rangle = \int \prod_{i=1}^{n} d\omega_i\, \omega_i^{\Delta_i-1}\, e^{-i\epsilon_i\omega_i u_i}\, \mathcal{S}_n$$

An example of a Carrollian primary

- Consider the modified Mellin transform of a plane wave in Minkowski space propagating along a null direction q^μ: $e^{\pm i\omega\, q\cdot x}$ in the parametrization discussed above

$$\Phi^{\pm}_{\Delta}(x^\mu\,|\,u,z,\bar{z}) = \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\pm i\omega u} e^{-\epsilon\omega}\, e^{\pm i\omega\, q\cdot x}$$

$$= \frac{(\mp i)^\Delta \Gamma(\Delta)}{(-q\cdot x - u \mp i\epsilon)^\Delta}$$

These wavefunctions satisfy $\Box\, \Phi^{\pm}_{\Delta}(x^\mu\,|\,u,z,\bar{z}) = 0$ for a fixed u. They transform as Carrollian primaries under bulk Lorentz transformations and translations (Banerjee [2018]).

- Modified Mellin transformation facilitates a basis change from plane waves (used to scatter in S-matrix) to conformal Carrollian primaries (used to construct operators within correlation functions).

Carroll 2 point function from the S-matrix

- The two point free theory Scattering amplitude is

$$\langle p_1, \sigma_1 | p_2 \sigma_2 \rangle = (2\pi)^3\, 2E_{p_1}\, \delta^{(3)}(\vec{p}_1 - \vec{p}_2)\, \delta_{\sigma_1+\sigma_2,0}$$

$$= 4\pi^3 \frac{\delta(\omega_1 - \omega_2)\, \delta^2(z_1 - z_2)}{\omega_1}\, \delta_{\sigma_1+\sigma_2,0}$$

The modified Mellin transformation of this amplitude is given by

$$4\pi^3 \delta_{\sigma_1+\sigma_2,0} \int_0^\infty d\omega_1\, d\omega_2\, \omega_1^{\Delta_1 - 1} \omega_2^{\Delta_2 - 1} e^{-i\omega_1 u_1} e^{i\omega_2 u_2}\, \frac{\delta(\omega_1 - \omega_2)\, \delta^2(z_1 - z_2)}{\omega_1}$$

$$= 4\pi^3 \Gamma(\Delta_1 + \Delta_2 - 2)\, \frac{\delta^2(z_2 - z_1)}{(i(u_1 - u_2))^{\Delta_1 + \Delta_2 - 2}} \delta_{\sigma_1+\sigma_2,0}$$

This result agrees with what one gets from the Carrollian Ward identities (Bagchi, Banerjee, Basu, Dutta [2022])

A vanilla boundary theory

- The holographic aspect: One can derive the correlator from an explicit boundary theory

$$S = \int du\, d^2z\, \frac{1}{2}(\partial_u \Phi(u, z, \bar{z}))^2$$

This theory is invariant under BMS_4 symmetries. The green's function for the theory is computed from

$$\partial_u^2 G(u - u', z^i - z'^i) = \delta^{(3)}(u - u', z^i - z'^i)$$

$$G(u - u', z^i - z'^i) = -\frac{i}{2}(u - u')\, \delta^{(2)}(z - z', z^i - z'^i)$$

For the massless Carrollian scalar field, $h, \bar{h} = \frac{1}{4}$: Matches with the two point function.The $\delta^{(2)}(z^i - z'^i)$ function makes the correlator ultra-local in the sphere directions. This is unlike the standard CFT 2-point function (which also respects the BMS_4 symmetries). Time-dependent correlation functions ~ Dynamics!

$$G(u, z, \bar{z}, u', z', \bar{z}'^i) = \frac{\delta_{h,h'}\, \delta_{\bar{h},\bar{h}'}}{(z - z')^{2h}\, (\bar{z} - \bar{z}')^{2\bar{h}}}$$

Flat limit of AdS/CFT

Recap: AdS in embedding space

AdS$_{d+1}$: $-(X^0)^2 - (X^1)^2 + \sum_{i=2}^{d+1} (X^i)^2 = - R^2$ is embedded in $\mathbb{R}^{1,1} \times \mathbb{R}^{1,d-1}$ (Ref: Penedones [2007])

$$ds^2 = - dX^+ dX^- - (dX^1)^2 + \sum_{i=2}^{d} (dX^i)^2$$

$$X^+ = - \frac{R(\cos\tau - \sin\rho\, \Omega_{d+1})}{\cos\rho}, \quad X^- = - \frac{R(\cos\tau + \sin\rho\, \Omega_{d+1})}{\cos\rho},$$

$$X^1 = - \frac{R\sin\tau}{\cos\rho}, \quad X^i = R\tan\rho\, \Omega_i\,, \qquad i = 2, \ldots, d$$

$$X^{\pm} = X^0 \pm X^{d+1}\,, \quad \Omega_i \in S^{d-1}$$

Recap: AdS in embedding space

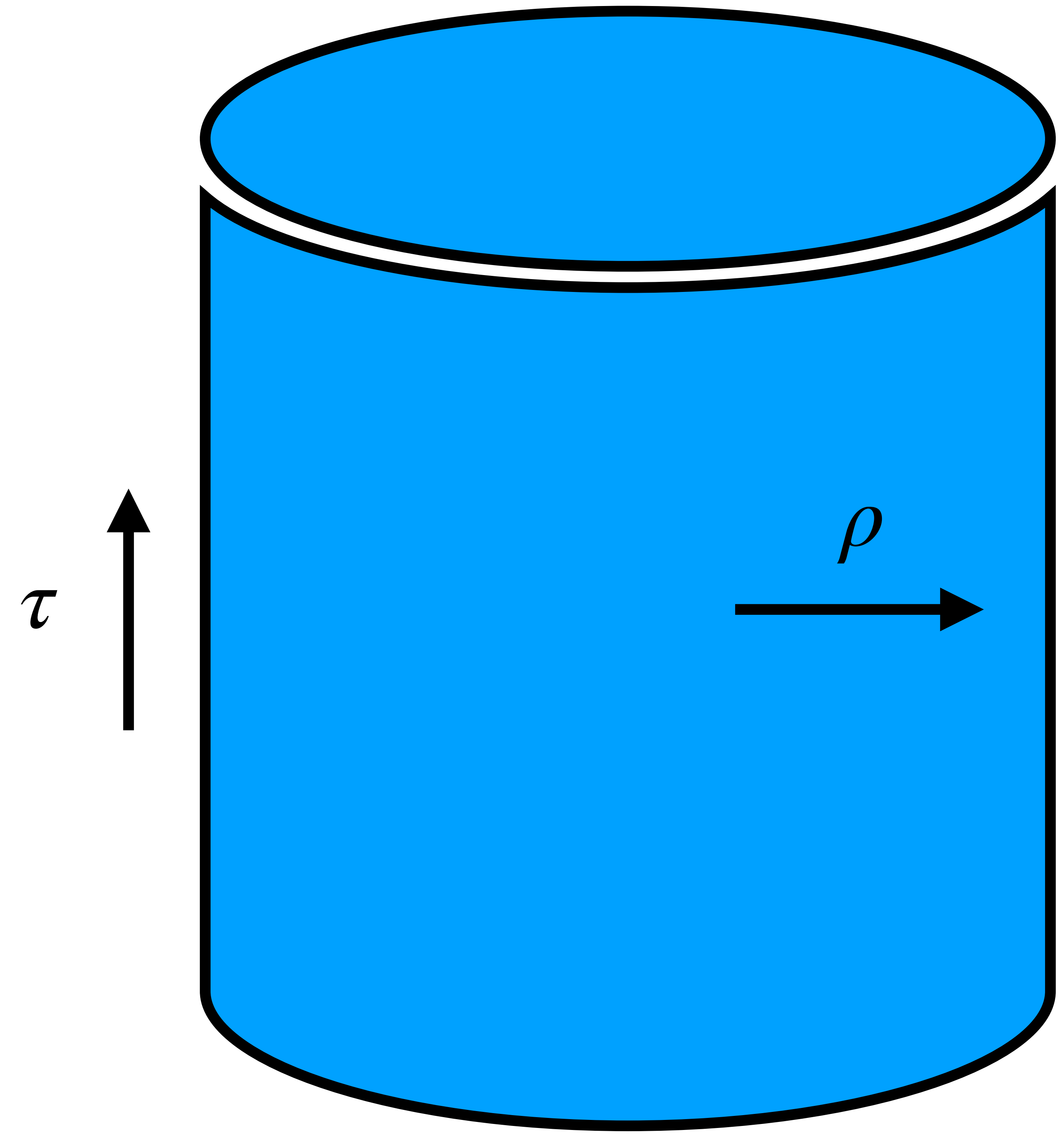
- The metric becomes ($\tau \in (-\infty, \infty)$ and $\rho \in [0, \frac{\pi}{2})$)

$$ds^2 = \frac{R^2}{\cos^2\rho}\left(-d\tau^2 + d\rho^2 + \sin^2\rho\, d\Omega^2_{S^{d-1}}\right)$$

- One can reach the boundary through (here $\mathbf{p}^2 = 0$)

$$\mathbf{p} = \lim_{\rho \to \frac{\pi}{2}} \frac{1}{2} R^{-1} \cos\rho\, \mathbf{X}$$

$$ds^2_{CFT} = -d\tau^2 + d\Omega^2_{d-1}$$



Flat limit in the bulk

- Implement the following rescaling (Giddings [1999])

$$\tau = \frac{t}{R}, \qquad \rho = \frac{r}{R}, \qquad R \to \infty$$

$$ds^2 = \frac{R^2}{\cos^2 \frac{r}{R}} \left(-\frac{dt^2}{R^2} + \frac{dr^2}{R^2} + \sin^2 \frac{r}{R}\, d\Omega^2_{S^{d-1}} \right)$$

$$ds^2 \xrightarrow{\lim_{R\to\infty}} -dt^2 + dr^2 + r^2 d\Omega^2$$

- In dimensionless coordinates, the limit is really $\tau \to 0$ and $\rho \to 0$, which effectively zooms into the centre of AdS.

Flat limit in the boundary

- We track the retarded time of the emergent flat space

$$u = t - r = R\,(\tau - \rho)$$

Fixing u as we let $t, r \to \infty$ leads to the null boundary of flat space. We aim to establish a relationship between u and AdS boundary time τ_p. The AdS boundary is reached by $\rho \to \dfrac{\pi}{2}$

$$u = R\left(\tau_p - \frac{\pi}{2}\right) \implies \tau_p = \frac{\pi}{2} + \frac{u}{R}$$

$$ds^2_{CFT} = -\frac{1}{R^2}du^2 + d\Omega^2 \quad \xrightarrow[c = \frac{1}{R} \to 0]{R \to \infty} \quad ds^2_{CFT} = 0.du^2 + d\Omega^2$$

CFT boundary metric

Null Carrollian metric

Celestial vs Carrollian perspectives

- de Gioia and Raclariu [2022] considered operator insertions at $\tau = \pm \frac{\pi}{2}$

$$\text{AdS Witten diagrams } \left(\tau = \pm \frac{\pi}{2}\right) \xrightarrow{R \to \infty} \text{Celestial Amplitudes}$$

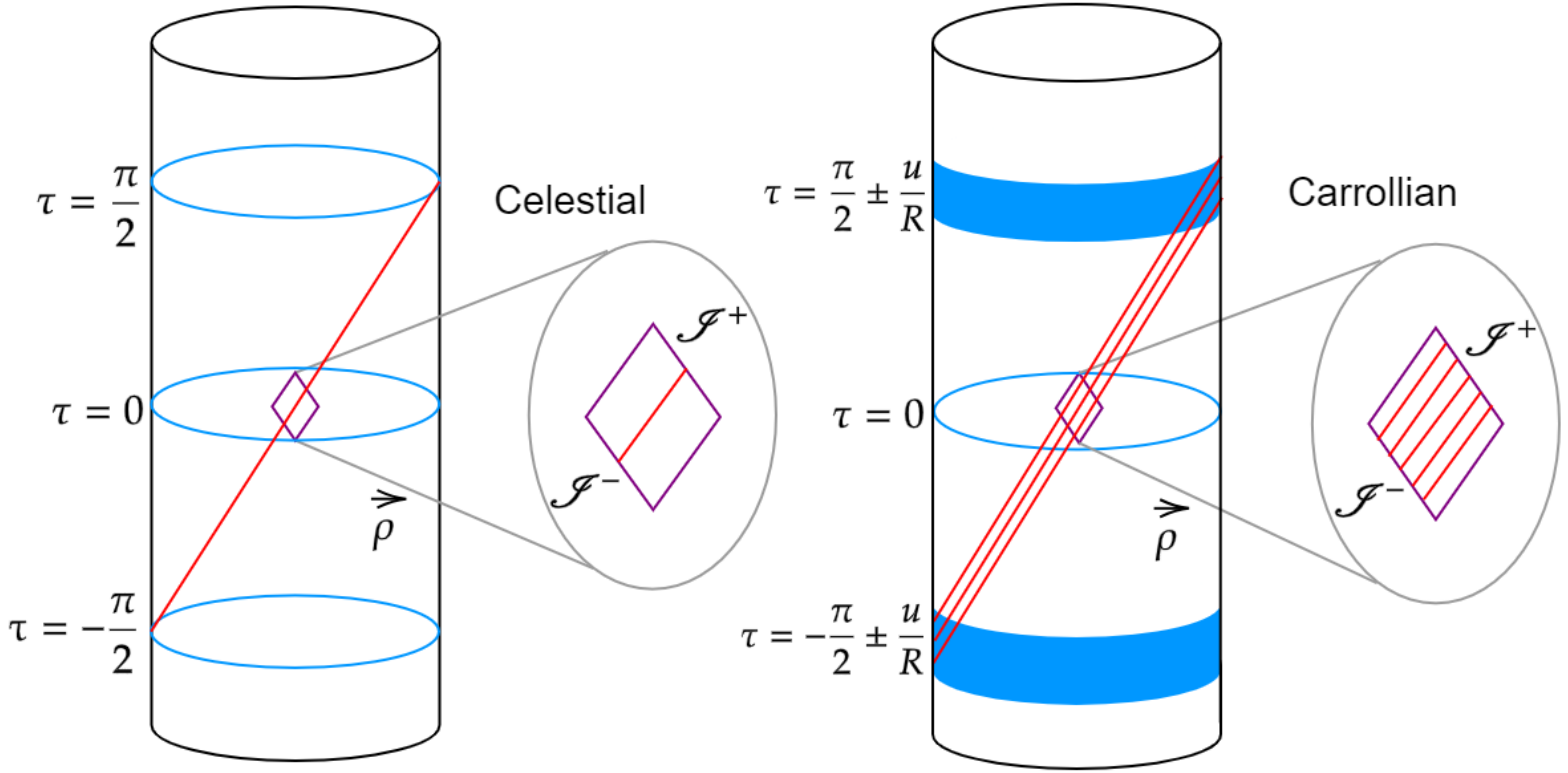
Spheres at $\tau = \pm \frac{\pi}{2}$ are antipodally identified.

- We (Bagchi, PD, Dutta [2303.07388]) consider a generalization of the boundary insertions that keeps track of the emergent null direction:

$$\text{AdS Witten diagrams } \left(\tau = \pm \frac{\pi}{2} + \frac{u}{R}\right) \xrightarrow{R \to \infty} \text{Carrollian Amplitudes}$$

Spheres at $\tau = \pm \frac{\pi}{2} + \frac{u}{R}$ are antipodally identified. This identification was later suggested by an analysis of CFT vector fields in the large R limit (de Gioia and Raclariu [2023]).

Celestial vs Carrollian perspectives



Bulk to boundary propagator is the conformal primary

- The bulk to boundary propagator $\mathbf{K}_{\Delta}(\mathbf{p}, \mathbf{x})$ in the embedding space is given by (Penedones [2010])

$$\mathbf{K}_{\Delta}(\mathbf{p}, \mathbf{x}) = \frac{C^d_{\Delta}}{(-2\mathbf{p}\cdot\mathbf{x} + i\epsilon)^{\Delta}}$$

Consider the bulk point $\mathbf{x}$ to be within the emergent flat diamond and the boundary point $\mathbf{p}$ to be within the band $\tau = +\frac{\pi}{2} + \frac{u}{R}$. In the large R expansion,

$$\mathbf{K}_{\Delta}(\mathbf{p}, \mathbf{x}) = C^d_{\Delta}\left(\frac{1}{(-u - \tilde{q}^{+}\cdot x + i\epsilon)^{\Delta}} + \mathcal{O}(R^{-1})\right) \qquad \text{(Outgoing)}$$

$x = (t, r\Omega)$, $\tilde{q}^{+} = (1, \Omega_{\mathbf{p}}) \in \mathbb{R}^{1,d}$ (i.e.) $\tilde{q}^{+}$ is a null vector in the direction of the boundary point $\mathbf{p}$ $(\tau_{\mathbf{p}}, \Omega_{\mathbf{p}})$

This is clearly the conformal Carrollian primary wave function we have seen before!

Bulk to boundary propagator is the conformal primary

- If however, the insertion point is at $\tau = -\dfrac{\pi}{2} + \dfrac{u}{R}$, we have

$$\mathbf{K}_\Delta(\mathbf{p}, \mathbf{x}) = C^d_\Delta \left(\frac{1}{(u + \tilde{q}^- \cdot x + i\epsilon)^\Delta} + \mathcal{O}(R^{-1}) \right) \qquad \text{(Ingoing)}$$

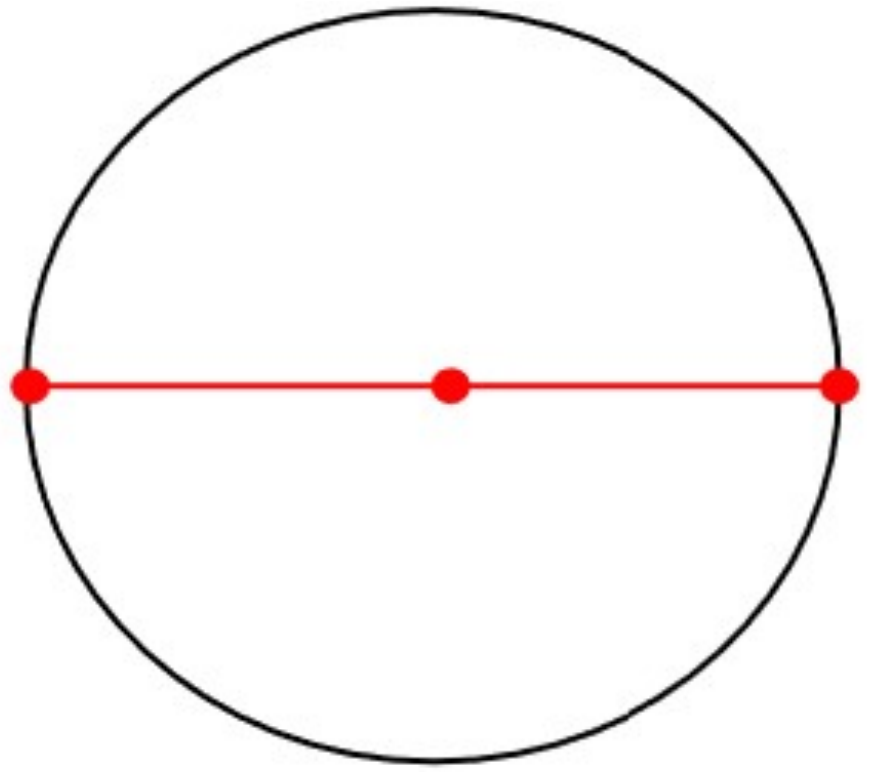
Where crucially $\tilde{q}^- = (1, \Omega^A_\mathbf{p})$ which is given by the antipodal matching condition

$$\boxed{\Omega^A_\mathbf{p} = -\Omega_\mathbf{p}}$$

- The bulk to boundary propagator in the large R limit is a Carrollian primary

$$\mathbf{K}_\Delta(\mathbf{p}, \mathbf{x}) = N^d_\Delta \int_0^\infty d\omega\, \omega^{\Delta-1} e^{\mp i\omega u} e^{\mp i\omega\, \tilde{q}^\pm \cdot x} e^{-\epsilon\omega}$$

Example: Scalar 2 point function



$$\langle O_{\Delta_1}(\mathbf{p}_1) O_{\Delta_2}(\mathbf{p}_2)\rangle = \int_{AdS_4} d^4\mathbf{x}\, \mathbf{K}_{\Delta_1}(\mathbf{p}_1, \mathbf{x})\, \mathbf{K}_{\Delta_2}(\mathbf{p}_2, \mathbf{x})$$

- Inserting $\mathbf{p}_1$ at $\tau_{\mathbf{p}} = -\dfrac{\pi}{2} + \dfrac{u_1}{R}$ (ingoing) and $\mathbf{p}_2$ at $\tau_{\mathbf{p}} = \dfrac{\pi}{2} + \dfrac{u_2}{R}$ (outgoing), we get the time dependent Carrollian correlation functions in the large R limit

$$\propto \int d^4x \int d\omega_1\, \omega_1^{\Delta_1 - 1}\, e^{i\omega_1 u_1}\, e^{i\omega_1 \tilde{q}^- \cdot x}\, e^{-\epsilon\omega_1} \int d\omega_2\, \omega_2^{\Delta_2 - 1}\, e^{-i\omega_2 u_2}\, e^{-i\omega_2 \tilde{q}^+ \cdot x}\, e^{-\epsilon\omega_2}$$

$$\propto \int d\omega_1\, d\omega_2\, \omega_1^{\Delta_1 - 1}\, \omega_2^{\Delta_2 - 1}\, e^{i\omega_1 u_1 - i\omega_2 u_2}\; e^{-\epsilon(\omega_1 + \omega_2)} \int d^4x\, e^{i(\omega_1 \tilde{q}^- - \omega_2 \tilde{q}^+)\cdot x}$$

Integration over x ensures momentum conservation ~ Ultra-locality

$$\langle O_{\Delta_1}(\mathbf{p}_1) O_{\Delta_2}(\mathbf{p}_2)\rangle = \mathscr{A} \frac{\delta^2(z_2 - z_1)}{(i(u_2 - u_1))^{\Delta_1 + \Delta_2 - 2}}\,, \qquad \mathscr{A} \sim \frac{\Gamma(\Delta_1 + \Delta_2 - 2)}{R^{2-(\Delta_1 + \Delta_2)}}$$

Kinematics of three point functions

- One cannot have momentum conservation in $\mathbb{R}^{1,3}$ with three null momenta. Suppose if you have $\delta^{(4)}(\omega_1\tilde{q}_1 + \omega_2\tilde{q}_2 - \omega_3\tilde{q}_3)$ with $\tilde{q}_i^2 = 0$,

$$(\omega_1\tilde{q}_1 + \omega_2\tilde{q}_2)^2 = 2\omega_1\omega_2\tilde{q}_1 \cdot \tilde{q}_2 \neq \omega_3^2\tilde{q}_3^2$$

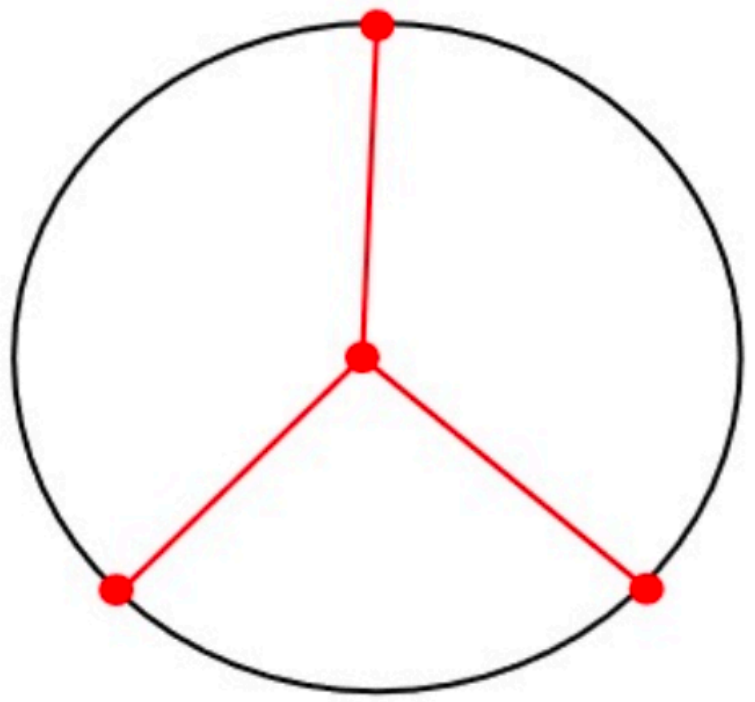
This is satisfied only when $\tilde{q}_1 \cdot \tilde{q}_2 = 0$ (collinear limit) or $\omega_{1,2} = 0$ (soft limit).

- Alternatively, we could also work in the split signature where z and $\bar{z}$ are independent (Pasterski, Shao, Strominger [2017]). This corresponds to complexifying the momenta

$$\delta^{(4)}(\omega_1\tilde{q}_1 + \omega_2\tilde{q}_2 - \omega_3\tilde{q}_3) = \frac{4}{\omega_3^2 z_{23} z_{31}}\delta\left(\omega_1 - \omega_3\frac{z_{32}}{z_{12}}\right)\delta\left(\omega_2 - \omega_3\frac{z_{31}}{z_{21}}\right)\delta(\bar{z}_{13})\delta(\bar{z}_{23})$$

Example: Scalar three point function (collinear limit)

- Lorentz invariant split of the delta function



$$\delta^{(4)}(\omega_1\tilde{q}_1 + \omega_2\tilde{q}_2 - \omega_3\tilde{q}_3) = \frac{1}{\omega_3^3}\,\delta(\omega_1 + \omega_2 - \omega_3)\,\delta(z_{12})\,\delta(\bar{z}_{12})\,\delta(z_{13})\,\delta(\bar{z}_{13})$$

$$\langle O_{\Delta_1}(\mathbf{p}_1)O_{\Delta_2}(\mathbf{p}_2)O_{\Delta_3}(\mathbf{p}_3)\rangle = \int_{AdS_4} d^4\mathbf{x}\,\mathbf{K}_{\Delta_1}(\mathbf{p}_1,\mathbf{x})\mathbf{K}_{\Delta_2}(\mathbf{p}_2,\mathbf{x})\mathbf{K}_{\Delta_3}(\mathbf{p}_3,\mathbf{x})$$

Inserting $\mathbf{p}_1$, $\mathbf{p}_2$ and $\mathbf{p}_3$ in the band around $\tau = \pm\dfrac{\pi}{2}$ and performing the integral over x

$$\mathscr{A}_{(3)}\,\delta^2(z_{12})\,\delta^2(z_{13})\sum_{k=0}^{\Delta_3-4}\frac{{}^{\Delta_3-4}C_k\,\Gamma(k+\Delta_1)\,\Gamma(\Delta_2+\Delta_3-k-4)}{(i\,u_{31})^{\Delta_1+k}\,(i\,u_{32})^{\Delta_2+\Delta_3-4-k}}$$

Agrees with the Carrollian Ward identities. See also (Nguyen [2023])

Example: Scalar 3 point function (soft limit)

- The Lorentz invariant split for the soft limit

$$\delta^{(4)}(\omega_1\tilde{q}_1 + \omega_2\tilde{q}_2 - \omega_3\tilde{q}_3) = \frac{1}{\omega_3^2}\frac{1}{z_{12}\bar{z}_{12}}\,\delta(\omega_2)\,\delta(\omega_1 - \omega_3)\,\delta(z_{13})\,\delta(\bar{z}_{13})$$

Due to the crucial soft factor $\delta(\omega_2)$, we get a restriction on Δ_2

$$\int_0^\infty d\omega_2\,\omega_2^{\Delta_2 - 1}\,e^{i\omega_2 u_2}\,\delta(\omega_2) = \delta_{\Delta_2,1}$$

$$\langle O_{\Delta_1}(\mathbf{p}_1)O_{\Delta_2}(\mathbf{p}_2)O_{\Delta_3}(\mathbf{p}_3)\rangle = \mathscr{A}_{(3)s}\,\frac{\Gamma(\Delta_1 + \Delta_3 - 3)\,\delta_{\Delta_2,1}\,\delta^2(z_{13})}{(i\,u_{13})^{\Delta_1+\Delta_3-3}\,z_{12}\,\bar{z}_{12}}$$

Generalization to spinning particles

- The bulk to boundary propagator of a spin J particle with dimension Δ is given by

$$K^{\Delta,J}_{\vec{\mu},\vec{\nu}}\left(\mathbf{p},\mathbf{x}\right)=C_{\Delta;J}\,\partial_{\mu_1}X^{A_1}\ldots\partial_{\mu_J}X^{A_J}\,\partial_{\nu_1}P^{B_1}\ldots\partial_{\nu_J}P^{B_J}\quad\frac{I_{\{A_1;\{B_1}\left(X;P\right)\ldots I_{A_J\};B_J\}}\left(X;P\right)}{\left(-P\cdot X+i\epsilon\right)^{\Delta}}$$

$$I_{A;B}\left(X;P\right)=\frac{-P\cdot X\eta_{AB}+P_AX_B}{-P\cdot X+i\epsilon}$$

A_i, B_i are the embedding space indices, μ_i denotes the bulk coordinates and ν_i denotes the boundary coordinates. $\{\ .\ \}$ denotes the symmetric traceless component. To implement the flat limit, we first rescale the μ_i and ν_i such that μ_i runs over $(t, r\Omega)$ and ν_i runs over (u, Ω) (de Gioia, Raclariu [2023]).

Example: Gluon 3 point function

- Spinning bulk to boundary propagator $K^{\Delta,1}_{\mu,\nu}(\mathbf{p},\mathbf{x})$ in the large R limit:

$$K^{\Delta,1}_{\mu,a} = C^d_{\Delta;1}\frac{\Delta-1}{\Delta}\frac{\pm\partial_a\tilde{q}_\mu}{(\mp u \mp \tilde{q}\cdot x + i\epsilon)^\Delta}$$

Gluon amplitude from loop corrections of the form F^3: $f^{abc}F^{a\mu}{}_{\nu}F^{b\nu}{}_{\rho}F^{c\rho}{}_{\mu}$. Three point function with the vertex factor (Bagchi, PD, Dutta [2311.11246]) is given below. See also (Salzer [2023]).

$$\langle O_{\Delta_1;\nu_1}(\mathbf{p}_1)O_{\Delta_2;\nu_2}(\mathbf{p}_2)O_{\Delta_3;\nu_3}(\mathbf{p}_3)\rangle = \int d^4\mathbf{x}\, K^{\Delta_1,1}_{\mu_1;\nu_1}(\mathbf{p}_1,\mathbf{x})\, K^{\Delta_2,1}_{\mu_2;\nu_2}(\mathbf{p}_2,\mathbf{x})\, K^{\Delta_3,1}_{\mu_3;\nu_3}(\mathbf{p}_3,\mathbf{x}) V^{\mu_1\mu_2\mu_3}_{3g}$$

$$\partial_{\bar{z}_1}\tilde{q}_{1\mu_1}\,\partial_{\bar{z}_2}\tilde{q}_{2\mu_2}\,\partial_{\bar{z}_3}\tilde{q}_{1\mu_3}\,V^{\mu_1\mu_2\mu_3}_{3g} \neq 0$$

$(---)$ amplitude

$$\langle O_{\Delta_1;\bar{z}_1}(\mathbf{p}_1)O_{\Delta_2;\bar{z}_2}(\mathbf{p}_2)O_{\Delta_3;\bar{z}_3}(\mathbf{p}_3)\rangle \propto f^{abc}\frac{z_{12}^{\Delta_3}\, z_{32}^{\Delta_1}\, z_{31}^{\Delta_2}\,\Gamma(\Delta_1+\Delta_2+\Delta_3-1)\,\delta(\bar{z}_{13})\,\delta(\bar{z}_{23})}{[-i(z_1u_{23}+z_2u_{31}+z_3u_{12})]^{\Delta_1+\Delta_2+\Delta_3-1}}$$

Conclusions

Conclusions

- We have argued that if one keeps track of the emergent null direction, AdS Witten diagrams naturally reduce to time dependent correlation functions in the leading order of the large AdS radius limit.

- The action of translations is more natural in the modified Mellin basis. As a result, graviton amplitudes are UV finite (Banerjee, Ghosh, Pandey, Saha [2019]).

- One can explicitly compute the $c \to 0$ limit of CFT correlation functions to arrive at these time dependent Carrollian correlators (Alday, Nocchi, Ruzziconi, Yelleshpur Srikant [2024])

- A differential representation for Carrollian correlators from the flat limit of the differential representation of AdS Witten diagrams (Chakrabortty, Hegde, Maurya [2024])

- Flat limit of ABJM (Lipstein, Ruzziconi, Yelleshpur Srikant [2025])

- Flat limit of AdS Witten diagrams in 3 dimensions and an analysis of the bulk point singularity (Surubaru, Zhu [2025])

Conclusions and closing comments

- Flat space limit of AdS Witten diagrams in general dimensions (Kulkarni, Ruzziconi, Yelleshpur Srikant [2025]) - See Romain's talk

- These works make use of the map between the S-matrix and Carrollian correlation functions. However, it is a work in progress to understand if these correlation functions are truly expectation values of operators acting on a Hilbert space.

- Quantum effects of Carroll theories have been studied in (Bagchi, Banerjee, Basu, Islam, Mondal [2022]; Mehra, Sharma [2023]; Banerjee, Basu, Krishnan, Maulik, Mehra, Ray [2023]; Figueroa-O'Farrill, Pérez, Prohazka [2023]; de Boer, Hartong, Obers, Sybesma, Vandoren [2023]; Chen, Sun, Zheng [2024]; Cotler, Jensen, Prohazka, Raz, Riegler, Salzer [2024]; Cotler, PD, Jensen [2025])

Thank you for your attention