Title. The connection between $\mathcal{R}$-boundedness and operator algebras

Abstract. A cornerstone result in harmonic analysis is the Mihlin multiplier theorem. In its simplest form, it states that every $m \in C^1(\mathbb{R} \setminus \{0\})$ satisfying

$$|m(\xi)| + |\xi m'(\xi)| \leq C, \qquad \xi \in \mathbb{R} \setminus \{0\}$$

defines a bounded Fourier multiplier operator on $L^p(\mathbb{R})$, i.e.

$$\left\|(m\hat{f})^\vee\right\|_{L^p(\mathbb{R})} \lesssim \|f\|_{L^p(\mathbb{R})}, \qquad f \in L^p(\mathbb{R}).$$

Now let X be a Banach space and suppose that $m\colon \mathbb{R} \setminus \{0\} \to \mathcal{L}(X)$. If all such m satisfying $\|m(\xi)\|_{\mathcal{L}(X)} + \|\xi m'(\xi)\|_{\mathcal{L}(X)} \leq C$ define a bounded Fourier multiplier operator on $L^p(\mathbb{R}; X)$, then X must be a Hilbert space.

In the seminal result of Weis, it was shown that one can obtain a vector-valued Mihlin multiplier theorem beyond the Hilbert space case if one imposes a stronger assumption on m. The uniform boundedness assumption needs to be replaced by a so-called $\mathcal{R}$-boundedness condition. Therefore, one could say that $\mathcal{R}$-boundedness of the range of m has some Hilbertian structure, leading to bounded Fourier multiplier operators. I will make this heuristic rigorous by representing $\mathcal{R}$-bounded families of operators on a Hilbert space.

This talk is based on joint worked with Nigel Kalton and Lutz Weis