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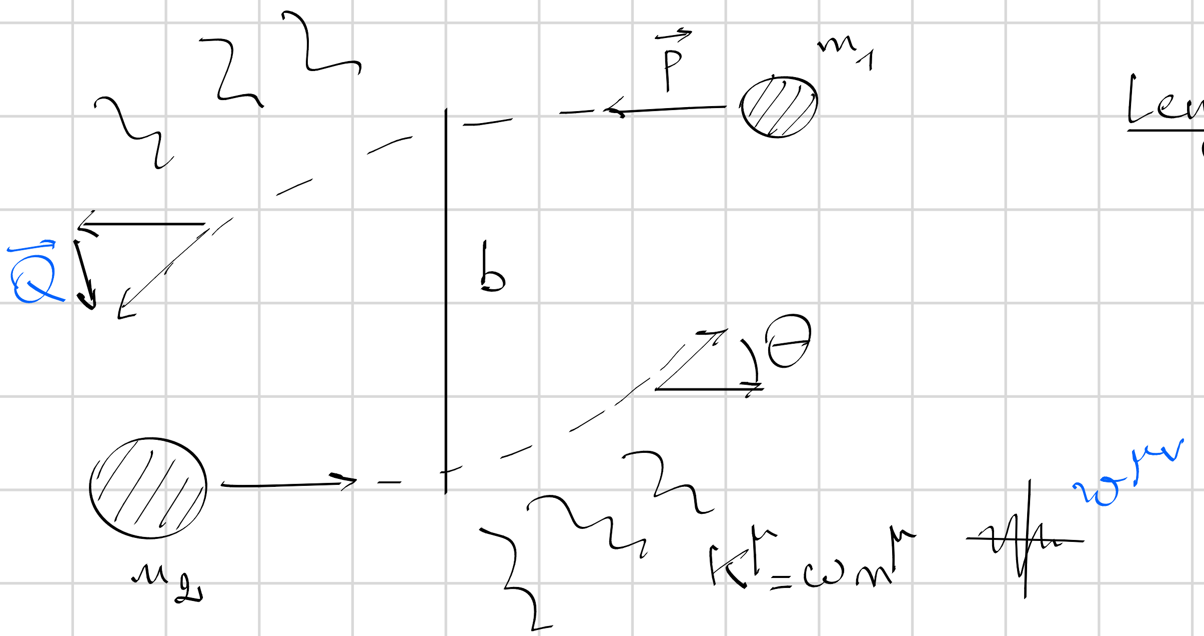
11/09/25
Edinburgh

Discussion / Vision Talk



Length scales: $\frac{\hbar}{m}$, $\boxed{(GE)\ Gm,\ b,\ \lambda = \frac{2\pi}{\omega}}$, r_{OBS}

TINY FOR CLASSICAL OBJECTS — HUGE

$\frac{Gm^2}{\hbar} \sim 10^{78}$

$\frac{\lambda}{r_{OBS}} \sim \frac{(c/10\,Hz)}{10^9\ ly} = \frac{c}{10^{10}\cdot\pi\cdot 10^7 \cdot c} \sim 10^{-17}$

(say $\frac{1}{10}$ roughly for early inspiral)

- Classical PM limit:

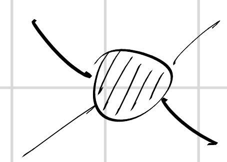
$\boxed{Gm \ll b \sim \lambda}$

$\boxed{\frac{Gm}{b} \sim Gm\omega \ll 1}$

nPM $\longleftrightarrow$ (n−1) LOOPS + 1 FT (b-space)

* $Q^\mu = Q^\mu_{1PM} + \dots + Q^\mu_{4PM} + \underline{Q^\mu_{5PM}} + \dots$

$\boxed{\frac{Q_{nPM}}{p} \sim \mathcal{O}\left(\left(\frac{Gm}{b}\right)^n\right)}$ $\longleftarrow$



– cuts (KMOC, eikonal ...)

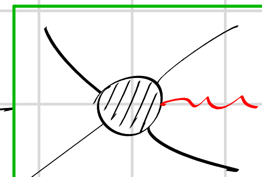
LATER Nathan Moynihan

* $h^{\mu\nu} \underset{\substack{r\to\infty \\ u,n\ \text{fixed}}}{\sim} \frac{4G}{r}\int \frac{d\omega}{2\pi}\, e^{-i\omega u}\, \tilde{w}^{\mu\nu}(\omega, n) + c.c.$

"Carroll" 1PF $\Delta = 1$, $D = 4$

$\tilde{w}^{\mu\nu} = \tilde{w}^{\mu\nu}_0 + \tilde{w}^{\mu\nu}_{1PM} + \tilde{w}^{\mu\nu}_{2PM} + \underline{\tilde{w}^{\mu\nu}_{3PM}} + \dots$

$\boxed{\frac{\tilde{w}_n}{\lambda p} \propto \mathcal{O}\left(\left(\frac{Gm}{b}\right)^n\right)}$ $\longleftarrow$



– cuts

$\propto \delta(\omega) \sum_{in} \frac{p_e^\mu p_e^\nu}{p_e\cdot n}$ "Liénard - Wiechert" STATIC FIELD

$4G w_{AB} = C_{AB}$ SHEAR

$\delta C_{AB} = T(n)\,\partial_u C_{AB} - \left[2 D_A D_B - \gamma_{AB}\Delta\right] T(n)$

SUPERTRANSLATION

$T = \sum_{in} 2G\, p_e\cdot n \log p_e\cdot n$

* P^μ_{rad}, $J^{\mu\nu}_{rad}$ from "$|\tilde{w}|^2$"

- Soft limit: $\boxed{Gm \sim b \ll \lambda}$ $\boxed{Gm\omega \sim b\omega \ll 1}$

$\frac{b\omega}{c} \sim \frac{10^6 m}{3\cdot 10^8 \frac{m}{s}}\,\omega = 10^{-3}\left(\frac{\omega}{1Hz}\right)$

so $\omega \sim 10$ Hz would be ok

Saha, Sahoo, Sen '18–'20

LATER Alok Laddha

$$\tilde{w}^{\mu\nu} \sim -\frac{i}{\omega}\sum_e \frac{p_e^\mu p_e^\nu}{p_e\cdot n} + G\log\omega\,(\checkmark) + G^2\omega(\log\omega)^2(\checkmark) + \dots$$ "LEADING" LOGS

$$+\, G\omega^0(\,?\,) + G^2\omega\log\omega(\,?\,) + \dots$$ "SUBLEADING" LOGS

$$\tilde{w}^{\mu\nu} \sim \omega^{2iGE\omega}\left[\frac{i}{\omega}a_0^{\mu\nu} - \sum_{n=1}^{\infty}(-i\omega)^{n-1}\frac{(\log\omega)^n}{n!}a_n^{\mu\nu} + \dots\right]$$ w/ $E = +\sum_{in} p_e\cdot n$

w/

$$a_0^{\mu\nu} = -\sum_e \frac{p_e^\mu p_e^\nu}{p_e\cdot n} \qquad a_1^{\mu\nu} = G\sum_{e,b}\frac{\tau_{eb}\,p_e^\mu}{p_e\cdot n}\,n_\rho\, p_{[b}^\rho p_{e]}^\nu$$

$$\left[\tau_{eb} = \frac{-\sigma_{eb}(2\sigma_{eb}^2-3)}{(\sigma_{eb}^2-1)^{\frac{3}{2}}}\right] \qquad a_2^{\mu\nu} = G^2\sum_{e,b,c}\frac{\tau_{eb}\,\tau_{ec}}{p_e\cdot n}\,n_\rho\, p_{[b}^\rho p_{e]}^\mu\, n_\sigma\, p_{[c}^\sigma p_{e]}^\nu$$

NOTICE $a_n^{\mu\nu} n_\nu = 0$

notably $a_0^{\mu\nu} n_\nu = 0$ by MOMENTUM CONSERVATION

* cross-checks for the regime $\boxed{Gm \ll b \ll \lambda}$ [everything works e.g. ~~[illegible]~~]

* go beyond i.e. use the combined limit to SIMPLIFY calculations and obtain explicit new ANALYTICAL predictions e.g. $\mathcal{O}(G\omega^0)$, $\mathcal{O}(G^2\omega\log\omega)$

$\rho = |\tilde{w}|^2$ (AMPLITUDE)^2 CALCULATION

* Nonlinear memory:

$$a_0^{\mu\nu} = -\sum_{em} \frac{p_{em}^{\mu} p_{em}^{\nu}}{p_{em}\cdot n} - \overbrace{\int \tilde{dk}\, \rho(k) \frac{k^\mu k^\nu}{k\cdot n}}^{a_{NL}^{\mu\nu}}$$

$$a_0^{\mu} n_\nu = -P_{in}^{\mu} - P_{out}^{\mu} - P_{rad}^{\mu} = 0$$

↪ collinear sing. goes away w/ TT

Explicit calculation @ $\mathcal{O}(G^3)$, doable at G^4...

* Rewriting à la Sahoo-Sen for $\log\omega$, $\omega(\log\omega)^2$: schematically,

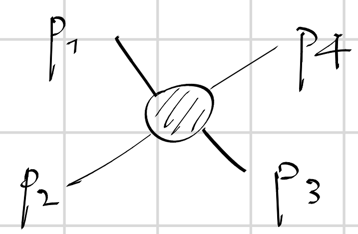
$$\sum_{ab,\dots} \longrightarrow \sum_{em, bm\dots} - \frac{P_{in}^{\mu} P_{in}^{\nu}}{P_{in}\cdot n} - \frac{P_{out}^{\mu} P_{out}^{\nu}}{P_{out}\cdot n}$$

Boschetti, Campiglia '25 $\longrightarrow$ log-translations

↪ bootstrap ?

• All-order Soft Theorem:

For $2\to 2$,



$$h(\sigma) = \frac{(2\sigma^2-3)\sigma}{(\sigma^2-1)^{3/2}}, \quad \sigma = -\frac{p_1\cdot p_2}{m_1 m_2}$$

$$\tilde{w}^{\mu\nu}_{LL} \sim -\frac{i}{E\omega}\, \omega^{2iGE\omega} \left[\left(\frac{p_1^\mu p_1^\nu}{p_1\cdot n} p_2\cdot n - p_1^\mu p_2^\mu + (1\leftrightarrow 2) \right) \omega^{iGEh(\sigma)\omega} - \left(\frac{p_3^\mu p_3^\nu}{p_3\cdot n} p_4\cdot n - p_3^\mu p_4^\nu + (1\leftrightarrow 2) \right) \omega^{-iGEh(\sigma)\omega} \right]$$

* derived for small $\sigma - 1$ — Alessio, Di Vecchia, CH '24

$\frac{m_1}{m_2}$ — Fucito, Morales, Russo '24

* EM result for generic # of particles (Karan, Khatun, Sahoo, Sen '25)
Magic resummation for $2\to 2$

$$g = \frac{q_1 q_2}{4\pi m_1 m_2 (\sigma^2-1)^{\frac{3}{2}}}$$

$$\tilde{a}^\mu = \frac{i}{E\omega}\left[\left(\frac{q_1 p_1^\mu}{p_1\cdot n}\, p_2\cdot n - q_1 p_2^\mu + (1\leftrightarrow 2)\right)\omega^{iEg\omega} + \left(\frac{q_4 p_4^\mu}{p_4\cdot n}\, p_3\cdot n - q_3 p_4^\mu + (1\leftrightarrow 2)\right)\omega^{-iEg\omega}\right]$$

- Scalar analog (Duary, Roy '25)
- WHY is $2\to 2$ magical
- Amplitude? Classical vs Quantum Logs
- Derivation / Understanding (Campiglia, Laddha '19; Agrawal, Donnay, Nguyen, Ruzziconi '23; Choi, Laddha, Puhm '24)

- High-Energy Limit: $E\longrightarrow\infty$ for $\frac{GE}{b}$ small ; $\lambda\sim b$ (?)

$$Q^\mu = Q^\mu_{1PM} + 0 + Q^\mu_{3PM} + ?$$

ACV90 singularity at fixed G $\longleftarrow$ P^μ_{rad}, $J^{\mu\nu}_{rad}$

$$\tilde{w}^{\mu\nu} = ?$$

"collinear" singularities at fixed G; RESUMMATIONS
Colferai, Ciafaloni, Veneziano; Gruzinov, Veneziano

$$\frac{dE}{d\omega} \longrightarrow G^3 \log\frac{GE}{b}$$ behavior explains the apparent singularity at fixed G