Wheeler-DeWitt equation and flat space holography

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Spatial infinity

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In AdS, information present in the bulk of a Cauchy slice is available near its boundary.

Holography in AdS

(picture taken from arXiv :2107.14802)

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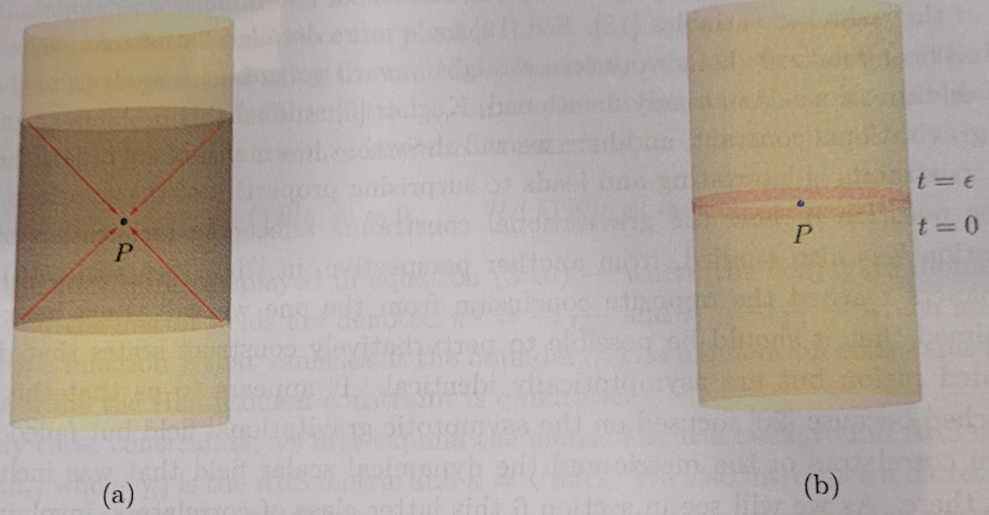

Figure 1: *A common misunderstanding of "holography" is that it only tells us that data on the timelike boundary of AdS can be used to reconstruct physics at the bulk point P as shown in the left subfigure. But this statement follows from the causal structure of AdS and does not require holography. Gravitational theories are genuinely holographic. In this paper we show how, in gravity, data on an infinitesimal time band (right subfigure) can be used to reconstruct physics in the bulk.*

between a component of the asymptotic metric and the energy of the state. This analysis also reveals how the unusual localization of quantum information in quantum gravity is visible at

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(or even the past of future null infinity)

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These key features hold both for asymptotically AdS or asymptotically flat spacetimes.

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Degeneracy of vacuum in asymptotically Minkowski space

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Degeneracy of vacuum in asymptotically Minkowski space (BMS group)

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(specific features of quantum gravity)

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Wheeler-DeWitt equation

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$$\hbar^2 G_{ijmn} \frac{\delta}{\delta g_{ij}} \left(\frac{\delta \Psi}{\delta g_{mn}} \right) + R\sqrt{g}\Psi = 0$$

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How does the BMS group act?

Known classically.

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These are all **boundary observables**, i.e., belong to the boundary algebra.

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which correspond to exact or non-exact operators.

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Spatial infinity is thus also a good place to understand holography.