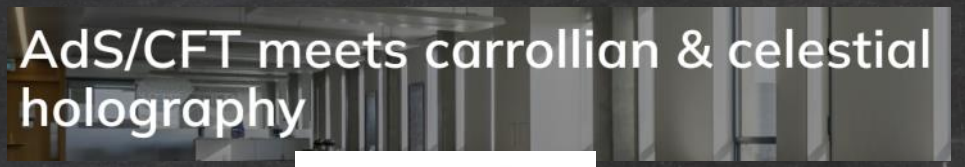
# Vision talk: amplitudes

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## Foundations & Dualities

- Modern fundamentals of amplitudes
- Colour–kinematics duality & double copy
- Ambitwistor strings

## Mathematical Structures

- Mathematical structures in Feynman integrals
- Multi-loop Feynman integrals
- Analytic bootstraps
- Positive geometry
- Integrability in fishnet theories

# Scattering Amplitudes

## Correlators & Strings

- Half-BPS correlators
- Modular covariance of type IIB ↔ $\mathcal{N}$=4 SYM

## Asymptotics & Limits

- Soft theorems & celestial amplitudes
- Multi-Regge limit

## Applications

- Collider physics
- Post-Minkowskian expansion
- Classical gravity from amplitudes

sagex amplitudes review 2022 topics (organised by chatGPT)

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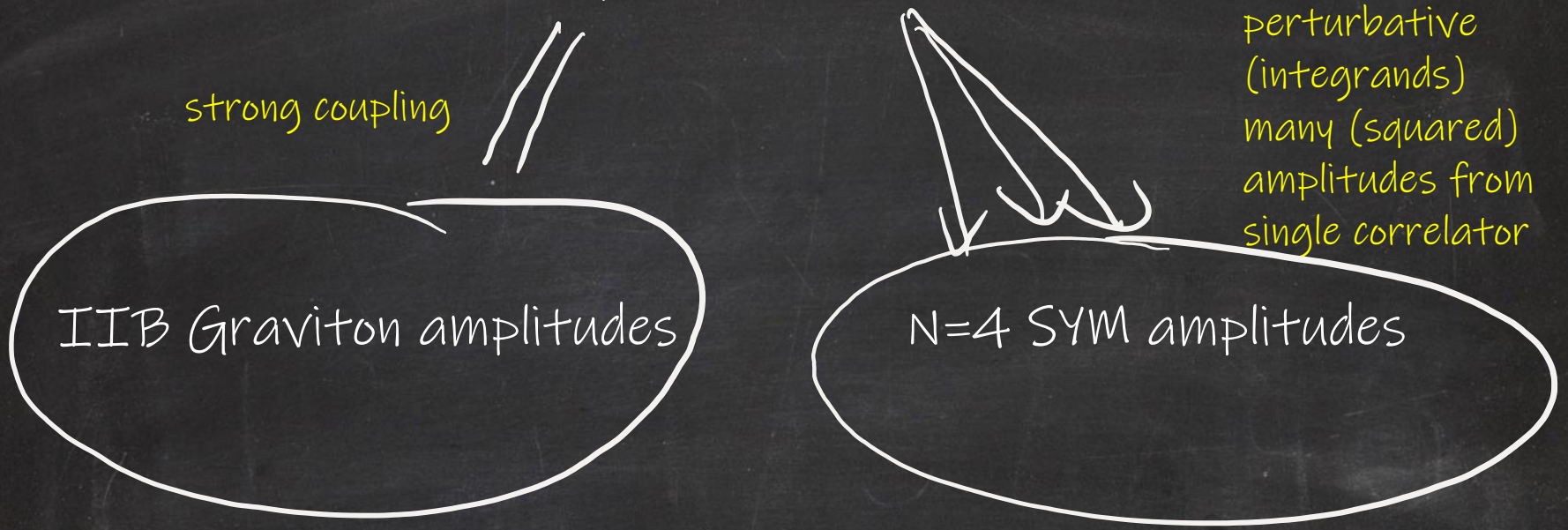
# N=4 SYM Half BPS correlators




- $\Phi(x,y) = \Phi_I y^I$, $y^{I=1..6}$, $y^I y^I = 0$
- Half BPS operators: $O_p(x,y) = Tr(\Phi(x,y)^p)$
- 4 point correlators: $\langle pqrs \rangle = \langle O_p O_q O_r O_s \rangle$
- $O_2$ dual to AdS gravity
- $O_p$ dual to higher S5 KK modes

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# Strong coupling

- 2000: Strong coupling, leading large N <2222> computed via AdS/CFT (tree level SUGRA).
- 2016: All <pqrs> (bootstrapped)
- direct uplift of <2222>

$$x_{ij}^2 \to x_{ij}^2 + y_{ij}^2$$

$$\mathcal{O}(x,y) = O_2(x,y) + O_3(x,y) + \cdots$$

master operator

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \langle 2222 \rangle \Big|_{x_{ij}^2 \to x_{ij}^2 + y_{ij}^2}$$

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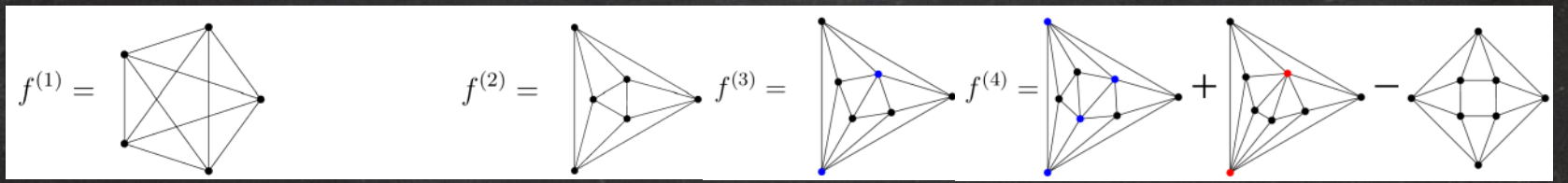
# Strong coupling

- 1, 2 loop quantum gravity corrections <2222> bootstrapped also (some <pqrs> at 1 loop, uplift not understood)
- tree level 5-point <pqrst> (Ellis talk)
- String corrections! towards Virasoro Shapiro in AdSxS
  - Tree-level (alpha')^8 known all <pqrs>
  - UPLIFT $x^2 \to x^2 + y^2$ (scalar effective action, AdSxS Witten diagrams -> origin? IIB SUGRA in terms of a scalar effective action?)
- first curvature correction to flat space all orders in alpha' (Virasoro-Shapiro) known, for all <pqrs> (Ellis talk)

"Higher: loops, string corrections, KK modes, # points"

# Weak coupling

- 2000: <2222> 2 loop
- Today: <pqrs> planar integrand to 12 loops !
- 4+l point permutation symmetry: f graphs; Graphical rules
- <pqrs>=<2222>$\Big|_{x_{ij}^2 \to x_{ij}^2 + y_{ij}^2}$

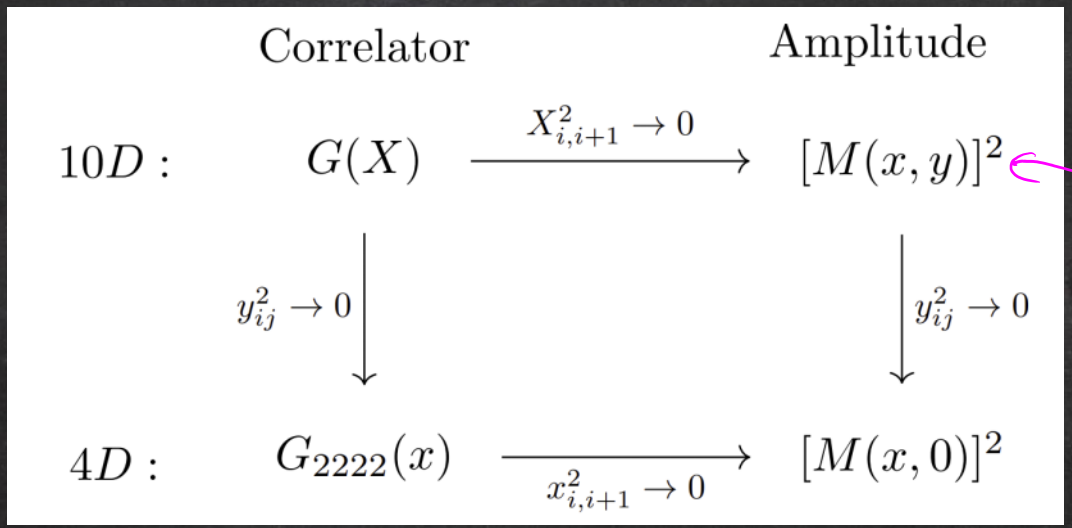



- Some higher points known, superconformal structure (WIP) (6,7 points to 2 loops, 5 to 3-loops) for <22222>
- Non-planar 4 loops <2222>
- <pqrst>?
- Integrals only known fully to 3 loops.....
- "Higher: loops, string corrections, KK modes, # points"

# Integrability

## Octagons, and amplitudes on the Coulomb branch

- Integrability: extended from anomalous dimensions to 3- and higher-point functions (hexagon)




Coulomb branch regulated amplitude.
No divergences

- M(x,y)= Octagon! ("simplest correlator", large R charge limit of <pqrs> determined at finite coupling by integrability)
- Implies v non trivial relations between integrals! (and periods)
- Higher points?

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# Correlator integrands from geometry

- Correlahedron: n-point l-loop correlator equivalent to geometry:
  Y in Gr(n+l, n+l+4)
  Xi in Gr(2, n+l+4):  $\langle Y\ Xi\ Xj \rangle > 0$
- Dual description: n twistor lines / conformal group

- Contains the amplituhedron (each correlahedron "contains" l+1 amplituhedra!)
- Recently verified to 4-point 4-loop
- Gives a pure basis of integrands
- Higher points?

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# Integrated correlators

- Localisation -> $\int \langle 2222 \rangle$ and $\int \langle 22pp \rangle$ exact all orders, both $\lambda$ ('t Hooft coupling) and N: new modular properties
- Integrate over the four external points (divided by the conformal group)
- $\int \langle pqrs \rangle$ (large N) as a function of $\lambda$ (not understood from localisation)

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# Bootstrap: beyond half BPS

- Many above results "bootstrapped"
- Numerical bootstrap: constrains correlators
- Technical issue to go further: general ( non half BPS ) correlators; superblocks
- SU(m,m|2n) universal structure:
  - Use SU(0,0|2n)= SU(2n) (finite reps) for SU(2,2|4) N=4 superconformal or even SU(2,2) conformal group
- Useful for 3-point functions too:
  eg 3 point conserved higher spin current tensor structures from finite rep theory. (WIP)

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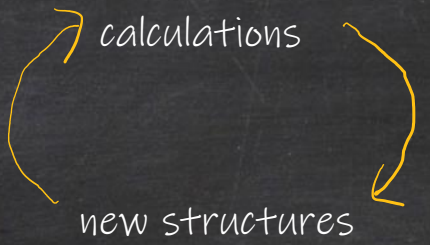
# Future vision

- "Higher: loops, string corrections, KK modes, # points"




- Many applications from limits of correlators: Amplitudes!; Energy Correlators; Form factor squared; Wilson loop correlators; < Wilson loop * Lagrangian >;
- Flat space limit -> Carollian of IIB / N=4 SYM: Concrete precise example + 10d structure, 9d Carollian (Srikant, Ruzziconi talk)
- AdS2xS2 -> 4d Carollian? (de Boer talk)
- Tensionless limit on AdS / free N=4 SYM? (Stieberger talk)
- Dream: Finite coupling (non integrated) correlators
- Ultimately: Want to Solve (sectors of) a 4d QFT (N=4 SYM)
- => Quantum gravity