Sandra Carillo, Sapienza University of Rome

Nonlinear evolution equations and Baecklund transformations

An overview on the study of nonlinear evolution equations of soliton type is provided.
In particular, 3rd order soliton equations as well as 5th order soliton equations are considered.
In addition non-commutative 3rd order counter parts and their links via Baecklund transformations are presented.

Notable analogies as well as discrepancies among the different cases are also stressed.

[1] C. Rogers, W.F. Shadwich, Baecklund transformations and their Appl.s, Accad. Press, 1982.
[2] S. C., M. Lo Schiavo, E. Porten, C. Schiebold, J. Math. Phys. 59, 4 (2018).
[3] S. C., B.Fuchssteiner, Physica A 166, 4 (1990).
[4] S. C., Appl. Num. Math. 141, 4 (2019).
[5] S. C., C. Schiebold, NODYCON 2019, W. Lacarbornara et. al Ed.s , (2020).
[6] S. C., M. Lo Schiavo, C. Schiebold, NODYCON 2020, W. Lacarbornara et. al Ed.s , (2020).
[7] S. C., C. Schiebold, NODYCON 2022, W. Lacarbornara et. al Ed.s , (2023).
[8] S. C., M. Lo Schiavo, C. Schiebold, Stud. Appl. Math. 154, 6 (2025).
[9] S. C., C. Schiebold, F. Zullo, Proceedings, MDPI, 2025; 123(1):9. 2025.

Vladimir Novikov, Loughborough University

Global Results in the Classification of Two-Component Integrable Evolutionary Systems

We consider the problem of classification of integrable hierarchies of two-component polynomial systems of evolutionary equations of odd orders. We derive the necessary and sufficient integrability conditions suitable to classify such integrable systems. We show that the classification relies on the classification of spectra - ratios of eigenvalues of the matrix of the linear part.

We prove that there are precisely 24 possible cases of spectra of integrable hierarchies. We obtain the complete list of S-integrable homogeneous hierarchies starting from orders 3 and 5. We relate the systems obtained in the result of the classification to the canonical hierarchies of Drinfeld-Sokolov.

Alexei Rybkin, University of Alaska Fairbanks

Inverse Scattering for KdV with Embedded Bound States

We study the inverse scattering problem for one-dimensional Schrödinger operators in the presence of embedded bound states and investigate their role in the dynamics of the Korteweg–de Vries (KdV) equation. While the inverse scattering transform is well understood for rapidly decaying or periodic potentials, significantly less is known for slowly decaying initial data, where spectral singularities and embedded eigenvalues may occur.

Focusing on a class of long-range potentials of Wigner–von Neumann type, we develop a framework that extends elements of scattering theory beyond the classical setting. In particular, we introduce appropriate spectral data that incorporate embedded bound states through additional norming constants and show that these states cannot be detected by the reflection coefficient alone.

Our main result provides a constructive transformation that converts resonances into embedded bound states while preserving key scattering features. This approach is closely related to binary Darboux transformations and is naturally adapted to the inverse scattering framework. As a consequence, we obtain explicit families of solutions to the KdV equation exhibiting bounded positon behavior and show that embedded bound states persist under the KdV flow.

These results contribute to the broader program of extending integrable systems techniques to nonstandard classes of initial data and offer new insight into the interplay between spectral theory and nonlinear wave propagation beyond the short-range regime.