Carroll theories from Lorentzian light-cone actions

Sucheta Majumdar

Centre de Physique Théoreique, Marseille

AdS/CFT meets Carrollian and Celestial Holography

ICMS Edinburgh, 8 -12 September 2025

Disclaimer!!!

What this talk **is not** about

- Holography: AdS/CFT, Carrollian or Celestial
- Gravity or String theory
- Scattering amplitudes

What this talk **is** about

Constructing Carrollian field theories from knowledge of Lorentzian ones

- Null reduction method [Duval, Gibbons, Horvathy, Zhang, 2014]
- Light-cone formulation

Disclaimer!!!

What this talk **is not** about

- Holography: AdS/CFT, Carrollian or Celestial
- Gravity or String theory
- Scattering amplitudes

What this talk **is** about

Constructing Carrollian field theories from knowledge of Lorentzian ones

- Null reduction method [Duval, Gibbons, Horvathy, Zhang, 2014]
- Light-cone formulation

Theme of the post-lunch session:

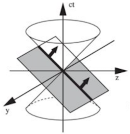
Hamiltonian techniques

Introduction

Light-cone formulation

- Light-cone coordinates in $(d+1)$ dimensions

$$x^+ = \frac{x^0 + x^{d+1}}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^{d+1}}{\sqrt{2}}, \quad x^i \, (i = 1, 2, \ldots, d-1)$$



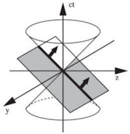
- Formulating QFTs using light-cone time x^+ [Dirac '49]

Introduction

Light-cone formulation

- Light-cone coordinates in $(d+1)$ dimensions

$$x^+ = \frac{x^0 + x^{d+1}}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^{d+1}}{\sqrt{2}}, \quad x^i \, (i = 1, 2, \ldots, d-1)$$



- Formulating QFTs using light-cone time x^+ [Dirac '49]

Why light-cone?

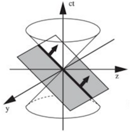
- Easier to solve constraints and eliminate redundant d.o.f. but with caution!

Introduction

Light-cone formulation

- Light-cone coordinates in $(d+1)$ dimensions

$$x^+ = \frac{x^0 + x^{d+1}}{\sqrt{2}}, \quad x^- = \frac{x^0 - x^{d+1}}{\sqrt{2}}, \quad x^i \, (i = 1, 2, \ldots, d-1)$$



- Formulating QFTs using light-cone time x^+ [Dirac '49]

Why light-cone?

- Easier to solve constraints and eliminate redundant d.o.f. but with caution!

  E.g. Light-cone gauge: $A_- = 0 \longrightarrow A_+ = f(A^i)$ and Maxwell Lagrangian $\mathcal{L}_{lc}[A^i]$

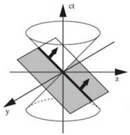
  Subtleties: Non-locality in x^- direction, **zero modes** of A_+, etc.

Introduction

Light-cone formulation

- Light-cone coordinates in $(d+1)$ dimensions

$$x^{+} = \frac{x^0 + x^{d+1}}{\sqrt{2}}, \quad x^{-} = \frac{x^0 - x^{d+1}}{\sqrt{2}}, \quad x^i \, (i = 1, 2, \dots, d-1)$$



- Formulating QFTs using light-cone time x^+ [Dirac '49]

Why light-cone?

- Easier to solve constraints and eliminate redundant d.o.f. but with caution!

  E.g. Light-cone gauge: $A_- = 0 \longrightarrow A_+ = f(A^i)$ and Maxwell Lagrangian $\mathcal{L}_{lc}[A^i]$

  Subtleties: Non-locality in x^- direction, **zero modes** of A_+, etc.

- Non-relativistic or Galilean aspects [Weinberg '66; Susskind'68]

  Light-cone Physics $\longleftrightarrow$ non-relativistic features

  $\longrightarrow$ 3D Galilei subgroup within 4D LC Poincaré

$$P_\mu P^\mu = P_+ P_- - P^i P_i = 0$$

$$\Rightarrow \text{Hamiltonian } P_+ = \frac{P^i P_i}{2P_-}$$

Many successes: QCD computations, String quantization, Scattering amplitudes, Self-dual actions, Higher spin theories...

THIS TALK

Goal: Use light-cone formulation to derive Carrollian field theories

Deriving d-dim Carroll theories

- Ultrarelativistic limit ($c \to 0$) of d-dim Poincaré theories

  [Levy-Leblond, Gomis, Hartong, Obers, Kleinschmidt, Duval, Gibbons, Horvathy, . . .]

  Carroll Hamiltonian actions: *necessary and sufficient conditions*
  [Henneaux, Salgado-Rebolledo, 2021]

Deriving d-dim Carroll theories

- Ultrarelativistic limit ($c \to 0$) of d-dim Poincaré theories

[Levy-Leblond, Gomis, Hartong, Obers, Kleinschmidt, Duval, Gibbons, Horvathy, . . .]

Carroll Hamiltonian actions: *necessary and sufficient conditions*
[Henneaux, Salgado-Rebolledo, 2021]

Electric and Magnetic Carroll sectors

Electric: $\mathcal{H}_E$ involve π or velocities $\dot{\phi}$

Magnetic: $\mathcal{H}_M$ involve spatial gradients $\partial_i \phi$

Deriving d-dim Carroll theories

- Ultrarelativistic limit ($c \to 0$) of d-dim Poincaré theories

  [Levy-Leblond, Gomis, Hartong, Obers, Kleinschmidt, Duval, Gibbons, Horvathy, . . .]

  Carroll Hamiltonian actions: *necessary and sufficient conditions*
  [Henneaux, Salgado-Rebolledo, 2021]

  Electric and Magnetic Carroll sectors

  Electric: $\mathcal{H}_E$ involve π or velocities $\dot{\phi}$

  Magnetic: $\mathcal{H}_M$ involve spatial gradients $\partial_i \phi$

- Intrinsically on d-dim Carroll manifolds using geometric objects: $g_{\mu\nu}$, $\mathfrak{n}^{\mu}$, etc.

  [Bergshoeff, Ciambelli, Gomis, Hartong, Obers, Oling Vandoren, Petropoulous,, . . .]

Deriving d-dim Carroll theories

- Ultrarelativistic limit ($c \to 0$) of d-dim Poincaré theories

  [Levy-Leblond, Gomis, Hartong, Obers, Kleinschmidt, Duval, Gibbons, Horvathy, ...]

  Carroll Hamiltonian actions: *necessary and sufficient conditions*
  [Henneaux, Salgado-Rebolledo, 2021]

  Electric and Magnetic Carroll sectors

  Electric: $\mathcal{H}_E$ involve π or velocities $\dot{\phi}$

  Magnetic: $\mathcal{H}_M$ involve spatial gradients $\partial_i \phi$

- Intrinsically on d-dim Carroll manifolds using geometric objects: $g_{\mu\nu}$, $\mathfrak{n}^\mu$, etc.

  [Bergshoeff, Ciambelli, Gomis, Hartong, Obers, Oling Vandoren, Petropoulous,, ...]

- Null reduction from $(d+1)$-dim Bargmann spacetimes [Duval, Gibbons, Horvathy, Zhang, 2014]

$$G = du \otimes dv + dv \otimes du + g_{ij} dx^i \otimes dx^j \,, \quad \mathfrak{n} = \partial_u$$

$\longrightarrow$ Galilean spacetime: Projection (KK reduction along $\mathfrak{n}$) [Julia-Nicolai '95]

$\longrightarrow$ Carrollian spacetime: Embedding (restricting to constant v surface)

THIS TALK

Goal: Apply null reduction to LC field theories

and derive same Carrollian theories obtained from $c \to 0$ method

Outline

- Light-cone Minkowski is *Bargmannian*
- Carroll theories from relativistic light-cone actions
- Some concluding remarks

Poincaré in light-cone coordinates

Minkowski metric

$$dS^2 = \eta^{lc}_{\mu\nu} dx^\mu dx^\nu = -2dx^+ dx^- + dx^i dx_i$$

A **flat** Bargmann structure:

$$\mathfrak{n} = \partial_+ , \quad \mathfrak{n}^\mu G_{\mu\nu} = 0 ,$$
$$\mathfrak{m} = \partial_- , \quad \mathfrak{m}^\mu G_{\mu\nu} = 0 ,$$

Poincaré in light-cone coordinates

Minkowski metric

$$dS^2 = \eta^{lc}_{\mu\nu} dx^\mu dx^\nu = -2dx^+ dx^- + dx^i dx_i$$

A **flat** Bargmann structure:

$$\mathfrak{n} = \partial_+ , \quad \mathfrak{n}^\mu G_{\mu\nu} = 0 ,$$
$$\mathfrak{m} = \partial_- , \quad \mathfrak{m}^\mu G_{\mu\nu} = 0 ,$$

Kinematical subgroups of LC Poincaré [**SM**, arXiv: 2406.10353] [Bagchi, Nachiketh, Soni (2024)]

$$\mathfrak{b}_+ = \{P_+, P_-, P_i, M_{ij}, M_{-i}\} \qquad \mathfrak{b}_- = \{P_+, P_-, P_i, M_{ij}, M_{+i}\}$$

x^+ Newtonian, x^- Carrollian $\qquad$ x^- Newtonian, x^+ Carrollian

- **Two copies** of d-dim Carroll, Bargmann, Galilei:

$$(\mathfrak{g}_+, \mathfrak{b}_+, \mathfrak{c}_+) \xleftrightarrow{x^+ \leftrightarrow x^-} (\mathfrak{g}_-, \mathfrak{b}_-, \mathfrak{c}_-)$$

- $\mathfrak{g}_+$: Galilei subgroup with light-cone Poincaré [Susskind '68]
- Carroll subgroups $\mathfrak{c}_\pm$: Stability group of light fronts at constant $x^\pm$

Outline

- Light-cone Minkowski is Bargmannian
- Carroll theories from relativistic light-cone actions
- Some concluding remarks

This talk

Consider x^+ (Carrollian) time and constant x^- hypersurface

Relevant subgroups: $\mathfrak{b}_- = \{P_+, P_-, P_i, M_{+i}, M_{ij}\}$ and $\mathfrak{c}_- = \{P_+, P_i, M_{+i}, M_{ij}\}$

Lorentzian LC action in $(d+1)$ dimensions

- Scalar field action:

$$\mathcal{S}[\phi,\dot{\phi}] \;=\; -\frac{1}{2}\int d^{d+1}x\,\eta^{lc}_{\mu\nu}\partial^\mu\phi\partial^\nu\phi \;=\; \int dx^+dx^-d^{d-1}x\,\left(\partial_+\phi\partial_-\phi-\frac{1}{2}\partial_i\phi\partial^i\phi\right)$$

- Hamiltonian formulation:

Conjugate momenta: $\pi=\dfrac{\delta\mathcal{L}}{\delta(\partial_+\phi)}=\partial_-\phi \quad\longrightarrow\quad$ not related to velocities $\partial_+\phi$

$\Rightarrow \quad$ Primary SCC : $\chi=\pi-\partial_-\phi$ $\qquad$ typical in LC theories

Canonical Hamiltonian $\quad \mathcal{H} \;=\; \pi\partial_+\phi-\mathcal{L} \;=\; \dfrac{1}{2}\partial_i\phi\partial^i\phi \quad\rightarrow\quad$ No ∂_+ or π terms in $\mathcal{H}$

Lorentzian LC action in $(d+1)$ dimensions

- Scalar field action:

$$\mathcal{S}[\phi,\dot{\phi}] \;=\; -\frac{1}{2}\int d^{d+1}x\; \eta^{lc}_{\mu\nu}\partial^\mu\phi\partial^\nu\phi \;=\; \int dx^+dx^-d^{d-1}x\left(\partial_+\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial^i\phi\right)$$

- Hamiltonian formulation:

Conjugate momenta: $\pi = \dfrac{\delta\mathcal{L}}{\delta(\partial_+\phi)} = \partial_-\phi \quad\longrightarrow$ not related to velocities $\partial_+\phi$

$\Rightarrow$ Primary SCC : $\chi = \pi - \partial_-\phi$ typical in LC theories

Canonical Hamiltonian $\mathcal{H} \;=\; \pi\partial_+\phi - \mathcal{L} \;=\; \dfrac{1}{2}\partial_i\phi\partial^i\phi \quad\rightarrow$ No ∂_+ or π terms in $\mathcal{H}$

Light-cone Hamiltonian action for scalars

$$\boxed{\mathcal{S}[\phi,\pi,\lambda] = \int dx^+dx^-d^{d-1}x\left\{\pi\partial_+\phi - \mathcal{H} - \lambda(\pi-\partial_-\phi)\right\}}$$

$${}^{(d+1)}\Omega = \int dx^-d^{d-1}x\; d_V\pi\wedge d_V\phi\,,\quad \{\pi(x),\phi(y)\}_{PB} = \delta(x^- - y^-)\delta^{d-1}(\mathbf{x}-\mathbf{y})$$

Lorentzian LC action in $(d+1)$ dimensions

- Scalar field action:

$$\mathcal{S}[\phi,\dot{\phi}] \;=\; -\frac{1}{2}\int d^{d+1}x\,\eta^{lc}_{\mu\nu}\partial^\mu\phi\partial^\nu\phi \;=\; \int dx^+dx^-d^{d-1}x\,\left(\partial_+\phi\partial_-\phi-\frac{1}{2}\partial_i\phi\partial^i\phi\right)$$

- Hamiltonian formulation:

Conjugate momenta: $\pi = \dfrac{\delta\mathcal{L}}{\delta(\partial_+\phi)} = \partial_-\phi \quad\longrightarrow$ not related to velocities $\partial_+\phi$

$\Rightarrow$ Primary SCC : $\chi = \pi - \partial_-\phi$ typical in LC theories

Canonical Hamiltonian $\mathcal{H} \;=\; \pi\partial_+\phi - \mathcal{L} \;=\; \dfrac{1}{2}\partial_i\phi\partial^i\phi \quad\rightarrow$ No ∂_+ or π terms in $\mathcal{H}$

Light-cone Hamiltonian action for scalars

$$\boxed{\mathcal{S}[\phi,\pi,\lambda] = \int dx^+dx^-d^{d-1}x\left\{\pi\partial_+\phi-\mathcal{H}-\lambda(\pi-\partial_-\phi)\right\}}$$

$${}^{(d+1)}\Omega = \int dx^-d^{d-1}x\;d_V\pi\wedge d_V\phi\,,\quad \{\pi(x),\phi(y)\}_{PB} = \delta(x^- - y^-)\delta^{d-1}(\mathbf{x}-\mathbf{y})$$

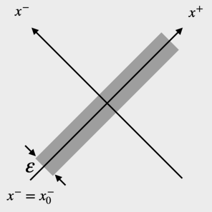
If π eliminated, we go back to $\mathcal{S}[\phi,\dot{\phi}] \rightarrow$ reduced phase space

$$\{\phi(x),\phi(y)\}_{DB} \sim \theta(x^- - y^-)\delta^{d-1}(\mathbf{x}-\mathbf{y})$$

Null reduction to d-dim Carroll theories

- Smearing function to restrict to constant x^- surface [Chen, Liu, Zheng, 2023]
  (Evaluate $\mathcal{S}$ around $x^- = x_0^- + \varepsilon$, then take $\varepsilon \to 0$)

$$\delta_\varepsilon(x - x_0^-) = \begin{cases} \frac{1}{\varepsilon}\,, & x_0^- - \frac{\varepsilon}{2} < x^- < x_0^- + \frac{\varepsilon}{2} \\ 0\,, & \text{otherwise} \end{cases}$$



Assuming x^- behaviour of the fields (Rescaling in $c \to 0$)

$$\phi\big|_{x^-=x_0^-} = \phi_m(x^+, x^i)\,, \quad \pi\big|_{x^-=x_0^-} = p_m(x^+, x^i)$$

- Carroll action:

$$\lim_{\varepsilon\to 0} {}^{(d+1)}\mathcal{S}_\varepsilon[\phi, \pi] = {}^{(d)}\mathcal{S}_{Carr}[\phi_m, \pi_m]$$

- **Only** magnetic Carroll scalars obtained

$$\boxed{{}^{(d)}\mathcal{S}_{Carr}[\phi_m, p_m] = \int dx^+ d^{d-1}x \left\{ p_m \partial_+ \phi_m - \frac{1}{2}\partial_i\phi_m\partial^i\phi_m \right\}}$$

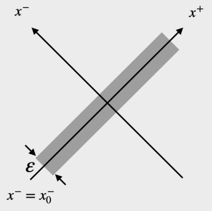
Null reduction to d-dim Carroll theories

- Smearing function to restrict to constant x^- surface [Chen, Liu, Zheng, 2023]
  (Evaluate $\mathcal{S}$ around $x^- = x_0^- + \varepsilon$, then take $\varepsilon \to 0$)

$$\delta_\varepsilon(x - x_0^-) = \begin{cases} \frac{1}{\varepsilon}\,, & x_0^- - \frac{\varepsilon}{2} < x^- < x_0^- + \frac{\varepsilon}{2} \\ 0\,, & \text{otherwise} \end{cases}$$

Assuming x^- behaviour of the fields (Rescaling in $c \to 0$)

$$\phi\big|_{x^-=x_0^-} = \phi_m(x^+, x^i)\,, \quad \pi\big|_{x^-=x_0^-} = p_m(x^+, x^i)$$



- Carroll action:

$$\lim_{\varepsilon\to 0} {}^{(d+1)}\mathcal{S}_\varepsilon[\phi, \pi] = {}^{(d)}\mathcal{S}_{Carr}[\phi_m, \pi_m]$$

- **Only** magnetic Carroll scalars obtained

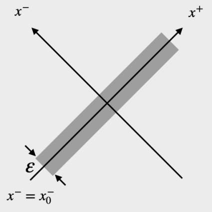
$$\boxed{{}^{(d)}\mathcal{S}_{Carr}[\phi_m, p_m] = \int dx^+ d^{d-1}x \left\{ p_m \partial_+ \phi_m - \frac{1}{2}\partial_i\phi_m\partial^i\phi_m \right\}}$$

- No way to get electric Carroll sector for *flat spacetimes* $\longrightarrow$ no π terms in $\mathcal{H}$
  Similar story for Galilean theories from null reduction
  [Julia, Nicolai '95] [Bergshoeff, Figueroa O'Farrrill, Gomis]

Null reduction to d-dim Carroll theories

- Smearing function to restrict to constant x^- surface [Chen, Liu, Zheng, 2023]
  (Evaluate $\mathcal{S}$ around $x^- = x_0^- + \varepsilon$, then take $\varepsilon \to 0$)

$$\delta_\varepsilon(x - x_0^-) = \begin{cases} \frac{1}{\varepsilon}, & x_0^- - \frac{\varepsilon}{2} < x^- < x_0^- + \frac{\varepsilon}{2} \\ 0, & \text{otherwise} \end{cases}$$



Assuming x^- behaviour of the fields (Rescaling in $c \to 0$)

$$\phi\big|_{x^-=x_0^-} = \phi_m(x^+, x^i), \quad \pi\big|_{x^-=x_0^-} = p_m(x^+, x^i)$$

- Carroll action:

$$\lim_{\varepsilon \to 0} {}^{(d+1)}\mathcal{S}_\varepsilon[\phi, \pi] = {}^{(d)}\mathcal{S}_{Carr}[\phi_m, \pi_m]$$

- **Only** magnetic Carroll scalars obtained

$$\boxed{{}^{(d)}\mathcal{S}_{Carr}[\phi_m, p_m] = \int dx^+ d^{d-1}x \left\{ p_m \partial_+ \phi_m - \frac{1}{2} \partial_i \phi_m \partial^i \phi_m \right\}}$$

- No way to get electric Carroll sector for *flat spacetimes* $\longrightarrow$ no π terms in $\mathcal{H}$
  Similar story for Galilean theories from null reduction
  [Julia, Nicolai '95] [Bergshoeff, Figueroa O'Farrrill, Gomis]

How to fix this?

Resolution [**SM**, Arxiv: 2507.03081]

- Deform $\mathcal{L}^{lc}[\phi, \dot{\phi}]$ to break Poincaré invariance by hand

  Pick a null vector: $\mathfrak{n} = \partial_+ \quad \Rightarrow \quad$ Relevant subgroup $\mathfrak{b}_- = \{P_+, P_-, P_i, M_{+i}, M_{ij}\}$

$$\mathcal{L}^{Barg}[\mathfrak{n}^\mu, \phi, \dot{\phi}] = \frac{1}{2}\alpha(\partial_+\phi)^2 + \partial_+\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial^i\phi$$

- Going to a general Bargmann spacetime

$$\eta^{lc}_{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix} \quad \longrightarrow \quad G_{\mu\nu} \begin{pmatrix} 0 & -1 & 0 \\ -1 & -\alpha & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix}$$

  Now, only $\mathfrak{n}^\mu$ lies in the kernel of G

- Change of coordinates to Bargmann LC coordinates $(x^+_\alpha, x^-_\alpha, x^i_\alpha)$

$$\begin{aligned} x^+_\alpha &= x^+ + \frac{\alpha}{2} x^- , \\ x^-_\alpha &= x^- , \\ x^i_\alpha &= x^i , \end{aligned}$$

Resolution [SM, Arxiv: 2507.03081]

- Deform $\mathcal{L}^{lc}[\phi, \dot{\phi}]$ to break Poincaré invariance by hand

  Pick a null vector: $\mathfrak{n} = \partial_+ \quad \Rightarrow \quad$ Relevant subgroup $\mathfrak{b}_- = \{P_+, P_-, P_i, M_{+i}, M_{ij}\}$

$$\mathcal{L}^{Barg}[\mathfrak{n}^\mu, \phi, \dot{\phi}] = \frac{1}{2}\alpha(\partial_+\phi)^2 + \partial_+\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial^i\phi$$

- Going to a general Bargmann spacetime

$$\eta^{lc}_{\mu\nu} = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix} \quad \longrightarrow \quad G_{\mu\nu} \begin{pmatrix} 0 & -1 & 0 \\ -1 & -\alpha & 0 \\ 0 & 0 & \delta_{ij} \end{pmatrix}$$

  Now, only $\mathfrak{n}^\mu$ lies in the kernel of G

- Change of coordinates to Bargmann LC coordinates $(x^+_\alpha, x^-_\alpha, x^i_\alpha)$

$$\begin{aligned} x^+_\alpha &= x^+ + \frac{\alpha}{2} x^- , \\ x^-_\alpha &= x^- , \\ x^i_\alpha &= x^i , \end{aligned}$$

Already known in LC quantization literature!!!
"Near light-front coordinates" [Lenz-Thies (1991)]

Resolution [**SM**, Arxiv: 2507.03081]

- Go to the Hamiltonian formulation

$$\mathcal{L}^{Barg} = \frac{1}{2}\alpha(\partial_+\phi)^2 + \partial_+\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial^i\phi$$

$$\pi = \alpha\partial_+\phi + \partial_-\phi\,, \quad \mathcal{H}_C^{Barg} = \frac{1}{2\alpha}(\pi - \partial_-\phi)^2 + \frac{1}{2}\partial_i\phi\partial^i\phi \quad \longrightarrow \quad \text{No SCCs}$$

$$\boxed{\mathcal{S}^{Barg} = \int dx^+ dx^- d^{d-1}x \left\{\pi\partial_+\phi - \mathcal{H}_C^{Barg}\right\}, \quad \Omega^{Barg} = \int dx^- d^{d-1}x\, d_V\pi \wedge d_V\phi}$$

Resolution [SM, Arxiv: 2507.03081]

- Go to the Hamiltonian formulation

$$\mathcal{L}^{Barg} = \frac{1}{2}\alpha(\partial_+\phi)^2 + \partial_+\phi\partial_-\phi - \frac{1}{2}\partial_i\phi\partial^i\phi$$

$$\pi = \alpha\partial_+\phi + \partial_-\phi\,, \quad \mathcal{H}_C^{Barg} = \frac{1}{2\alpha}(\pi - \partial_-\phi)^2 + \frac{1}{2}\partial_i\phi\partial^i\phi \quad \longrightarrow \quad \text{No SCCs}$$

$$\boxed{\mathcal{S}^{Barg} = \int dx^+ dx^- d^{d-1}x \left\{\pi\partial_+\phi - \mathcal{H}_C^{Barg}\right\}, \quad \Omega^{Barg} = \int dx^- d^{d-1}x\, d_V\pi \wedge d_V\phi}$$

- Two possibilities that preserve canonical structure

Magnetic Carroll sector

$$\phi \to \phi_m, \quad \pi = \partial_-\phi \to p_m, \quad \alpha \to \alpha/\varepsilon^2$$

$$\boxed{\mathcal{S}_M^{Carr} = \int dx^+ d^{d-1}x \left(p_m\partial_+\phi_m - \frac{1}{2}\partial_i\phi_m\partial^i\phi_m\right)}$$

EOM: $\partial_+\phi_m = 0\,, \quad \partial_+ p_m = \partial^i\partial_i\phi_m$

→ Momenta not related to velocities

Electric Carroll sector

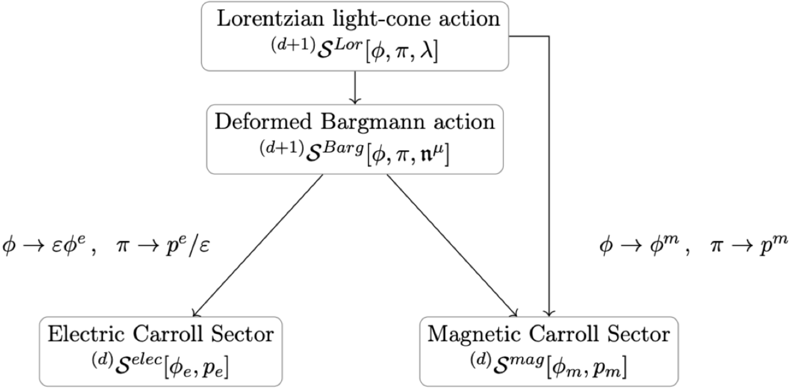
$$\phi \to \varepsilon\phi_e, \quad \pi \to p_e/\varepsilon, \quad \alpha \to \alpha/\varepsilon^2$$

$$\boxed{\mathcal{S}_E^{Carr} = \int dx^+ d^{d-1}x \left(p_m\partial_+\phi_m - \frac{1}{2\alpha}p_e^2\right)}$$

EOM: $\alpha\partial_+\phi_m = p_e\,, \quad \partial_+ p_m = 0$

→ Momenta related to velocities

Deformation essential for electric Carroll sector



Proof of "Carroll-ness"

Carroll hypersurface deformation:

$$G[\xi^+, \xi^i] = \int d^\perp x \, (\xi^+ \mathcal{H}^C + \xi^i \mathcal{P}_i^C) \,, \quad \xi^+ = b_i x^i + a^+ \,, \quad \xi^i = \omega^i_j x^j + a^i$$

- Carroll commutation relations hold

$$\Big[\mathcal{H}^C(x), \, \mathcal{H}^C(y)\Big] \; = \; 0 \,, \quad \Big[\mathcal{H}^C(x), \, \mathcal{P}_i^C(y)\Big] \; = \; \dots \,, \quad \Big[\mathcal{P}_i^C(x), \, \mathcal{P}_j^C(y)\Big] \; = \; \dots$$

  (for both electric or magnetic cases)

- Carroll transformations

$$\delta_{\xi^+, \xi^i} \phi^C = \{\phi^C, G[\xi^+, \xi^i]\} \,, \quad \delta_{\xi^+, \xi^i} p^C = \{p^C, G[\xi^+, \xi^i]\}$$

  render the Carrollian actions, $\mathcal{S}_M$ and $\mathcal{S}_E$, invariant

Proof of “Carroll-ness”

Carroll hypersurface deformation:

$$G[\xi^+,\xi^i] = \int d^\perp x \, (\xi^+\mathcal{H}^C + \xi^i\mathcal{P}_i^C)\,, \quad \xi^+ = b_i x^i + a^+\,, \quad \xi^i = \omega^i_j x^j + a^i$$

- Carroll commutation relations hold

$$\Big[\mathcal{H}^C(x),\, \mathcal{H}^C(y)\Big] \;=\; 0\,, \quad \Big[\mathcal{H}^C(x),\, \mathcal{P}_i^C(y)\Big] \;=\; \dots\,, \quad \Big[\mathcal{P}_i^C(x),\, \mathcal{P}_j^C(y)\Big] \;=\; \dots$$

  (for both electric or magnetic cases)

- Carroll transformations

$$\delta_{\xi^+,\xi^i}\phi^C = \{\phi^C, G[\xi^+,\xi^i]\}\,, \quad \delta_{\xi^+,\xi^i}p^C = \{p^C, G[\xi^+,\xi^i]\}$$

  render the Carrollian actions, $\mathcal{S}_M$ and $\mathcal{S}_E$, invariant

  Carroll Hamiltonian actions: **necessary and sufficient conditions** [Henneaux, Salgado-Rebolledo]

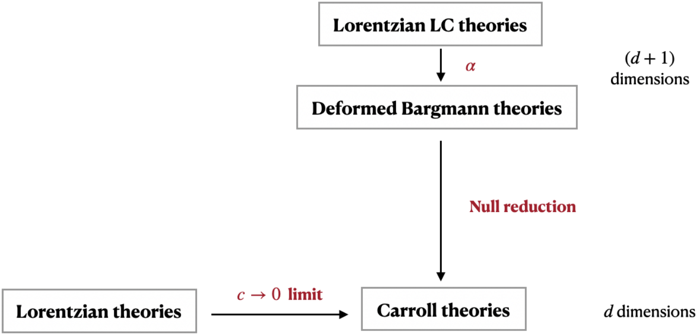
- Additionally, under Carroll boosts b^i,

$$\text{magnetic:} \quad \delta_b\phi_m = 0\,, \quad \delta_b p_m \neq 0\,.$$
$$\text{electric:} \quad \delta_b\phi_e \neq 0\,, \quad \delta_b p_e = 0\,.$$

Same theories, different means

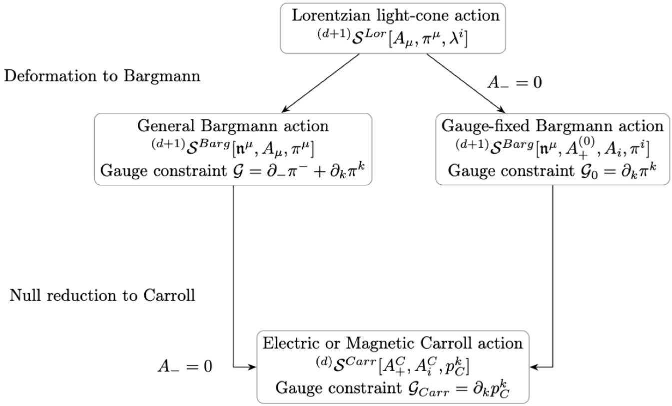
Starting point: Two very different Hamiltonian actions for Lorentzian FTs

$\rightarrow$ Constraints, IVP, Phase space, etc.



- Equivalence of Carroll theories obtained from two different methods
- Works for EM, Yang-Mills, p-form fields, etc. [**SM**, Arxiv: 2507.03081]

Light-cone gauge-fixing



Key observations:

- Does not matter when LC gauge is implemented
- : $A_+^{(0)}$: Zero mode as Largange multiplier for zero-mode gauge constraint $\mathcal{G}_0$ (LGTs)

General p-form gauge fields

- Lorentzian action for p-form field on $Mink^{d+1}$

$$A = \frac{1}{p!} A_{\mu_1 \dots \mu_p} dx^{\mu_1} \wedge \dots \wedge dx^{\mu_p}\,, \quad F = dA\,, \quad \mathcal{S}^{lc} = \int_M F^2$$

- Deformed Bargmann action

$$\mathcal{S}^{Barg} = \int_M F^2 + \alpha \int_M \mathfrak{n}^{\mu_1} \mathfrak{n}^{\nu_1} \dots \eta^{\mu_{p+1} \nu_{p+1}} F_{\mu_1 \dots \mu_{p+1}} F_{\nu_1 \dots \nu_{p+1}}$$

- Null reduction to Carroll:

Restrict to $x^- = x^0_- + \varepsilon$, then take $\varepsilon \to 0$

Set 'minus' component to zero: $F_{-\mu_1 \dots \mu_p} = 0$ Light-cone gauge

$$S^{Carr} = \lim_{\varepsilon \to 0} \mathcal{S}^{Barg} = \int_C F^2$$

Magnetic Carroll sector

$$F_{\mu_1 \dots \mu_{p+1}} \longrightarrow F_{i_1 \dots i_{p+1}}$$

Scalars: $F_\mu \longrightarrow F_i = \partial_i \phi$

EM: $F_{\mu\nu} \longrightarrow F_{ij} \sim \epsilon_{ijk} B_k$

Electric Carroll sector

$$F_{\mu_1 \dots \mu_{p+1}} \longrightarrow F_{+i_1 \dots i_p}$$

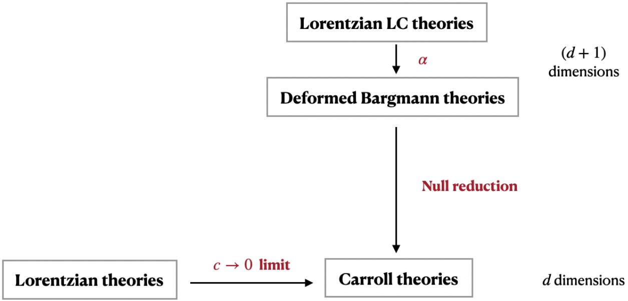
Scalars: $F_\mu \longrightarrow F_+ = \partial_+ \phi$

EM: $F_{\mu\nu} \longrightarrow F_{+i} \sim E_i$

Outline

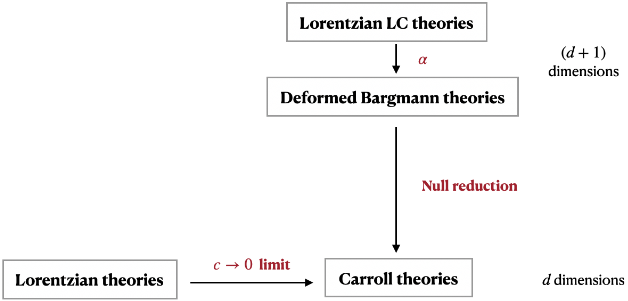
- Light-cone Minkowski is Bargmannian
- Carroll theories from relativistic light-cone actions
- Summary and Outlook

Summary and Outlook



- To-do list: Fermions [WIP], Curved geometries, Gravity ...
  (Double-null foliation [d'Inverno-Smallwood '80])

Summary and Outlook

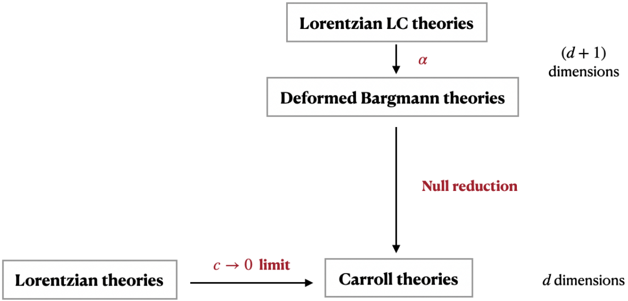


- To-do list: Fermions [WIP], Curved geometries, Gravity ...
  (Double-null foliation [d'Inverno-Smallwood '80])
- For Lorentzian QFTs: hard and cumbersome but possible

  Equal-time Quantization = Light-front Quantization

  (time t or x^0)

Summary and Outlook



- To-do list: Fermions [WIP], Curved geometries, Gravity ...

  (Double-null foliation [d'Inverno-Smallwood '80])

- For Lorentzian QFTs: hard and cumbersome but possible

  Equal-time Quantization $=$ Light-front Quantization

  (time t or x^0)

  Lessons for Quantum Carroll theories?

  Do quantum properties derived from the two methods agree with each other?

Summary and Outlook

- Discrete LC quantization (DLCQ) and other techniques

  $\rightarrow$ Vacuum structure

  $\rightarrow$ Explicit and Spontaneous symmetry breaking

  $\rightarrow$ IR divergences, etc.

  Examples: $\mathbb{Z}_2$ symmetry in ϕ^4, $O(N)$ sigma model, chiral symmetry breaking, ...

Summary and Outlook

- Discrete LC quantization (DLCQ) and other techniques

  $\rightarrow$ Vacuum structure

  $\rightarrow$ Explicit and Spontaneous symmetry breaking

  $\rightarrow$ IR divergences, etc.

  Examples: $\mathbb{Z}_2$ symmetry in ϕ^4, $O(N)$ sigma model, chiral symmetry breaking, ...

Review on Light-front Quantization [M. Burkardt, hep-ph/9505259]

Summary and Outlook

- Discrete LC quantization (DLCQ) and other techniques

  $\rightarrow$ Vacuum structure

  $\rightarrow$ Explicit and Spontaneous symmetry breaking

  $\rightarrow$ IR divergences, etc.

  Examples: $\mathbb{Z}_2$ symmetry in ϕ^4, $O(N)$ sigma model, chiral symmetry breaking, ...

Review on Light-front Quantization [M. Burkardt, hep-ph/9505259]

Summary, Conclusions and Outlook

LF field theory is a very promising approach toward calculating correlation functions along a light-like direction. Such correlation functions appear in the theoretical analysis of a variety of hard scattering processes, such as deep inelastic lepton-hadron scattering and asymptotic form factors. Probably the most intriguing and controversial property of LF Hamiltonians is