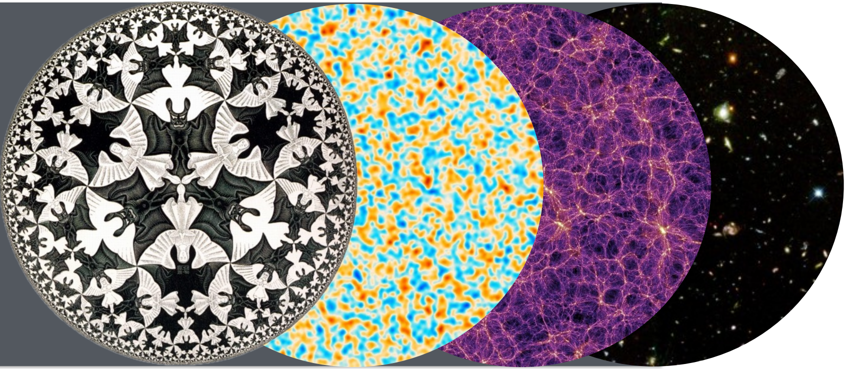
The renormalisation of IR divergences in de Sitter



Paul McFadden (Durham)

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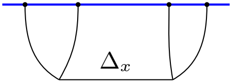
Introduction

IR divergences commonly arise in de Sitter in-in correlators, even at *tree level.*

For example, for 3- and 4-point functions of massless ($\Delta = 3$) and conformally coupled ($\Delta = 2$) scalars, the degrees of divergence ϵ^{-n} in dim. reg. are:

External Δ_i	3-points
$[222]$	0
$[322]$	1
$[332]$	0
$[333]$	1



	4-points		
External Δ_i	Contact	$\Delta_x = 2$	$\Delta_x = 3$
$[22;22]$	0	0	2
$[32;22]$	0	1	1
$[33;22]$	1	1	2
$[32;32]$	1	2	1
$[33;32]$	0	1	1
$[33;33]$	1	1	2

Introduction

These divergences arise from integrating over the vertices at late times and reflect the infinite volume of spacetime.

In the first part of this talk:

We show how to renormalise the Schwinger-Keldysh path integral for de Sitter correlators by adding local counterterms at the future boundary of de Sitter.

- While familiar from AdS/CFT, this approach has not been applied to dS.
- As a first step, we restrict here to tree level. At loop level, both UV and IR divergences arise, and their effects need to be carefully disentangled.

  In AdS: [Bañados, Bianchi, Muñoz, Skenderis '22]

- IR divergences in dS have also been intensively studied using a non-perturbative 'stochastic' formalism pioneered by Starobinsky in the 80s.

  For recent work, see, e.g., [Gorbenko & Senatore '19]

Introduction

Our renormalisation procedure is inspired by, but *independent* of holography.

In the second part of the talk, we draw a comparison:

- We show how the renormalisation of late-time fields in dS is analogous to the renormalisation of sources in AdS/CFT.
- However, renormalisation in dS is simpler than in AdS. Unlike in AdS, there are no conformal anomalies in dS.
- In both AdS and dS, the IR divergences of the bulk theory are local.

  This nontrivial property is *required* for the existence of a holographic duality with a local CFT.

  Local IR divergences in the bulk should map to local UV divergences of the dual CFT, providing new constraints on dS holography.

References

Work with Adam Bzowski and Kostas Skenderis:

- Renormalisation of IR divergences & holography in dS [JHEP 05 (2024) 053]
- A handbook of holographic 4-point functions [JHEP 12 (2022) 039]

Complete results for all 48 renormalised correlators of massless and conformal scalars, at up to 4-points, in both AdS and dS.

See also: Bzowski, Handbook of derivative AdS amplitudes [JHEP 04 (2024) 082]

Other recent works: [Benincasa & Vazão '24], [Wang, Pimentel & Achúcarro '22], ...

Set-up

- We consider tree-level correlators of light scalar fields ($d/2 < \Delta_i < d$), with polynomial interactions, on fixed $(d+1)$-dimensional de Sitter:

$$\mathrm{d}s^2 = \frac{L_{dS}^2}{\tau^2}[-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2]$$

- Instead of a late-time cut-off, we use dimensional regularisation:

$$d \to d + 2\epsilon, \qquad \Delta_i \to \Delta_i + \epsilon$$

  where the masses $m_i^2 L_{dS}^2 = \Delta_i(d - \Delta_i)$ with d the boundary dimension.

  The late-time asymptotics as $\tau \to 0^-$ are then power-law:

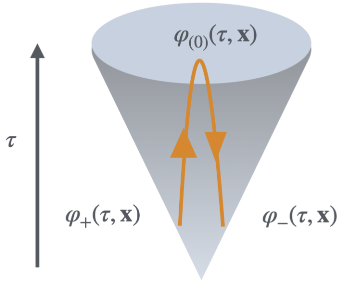
$$\varphi^i(\tau, \boldsymbol{x}) = (-\tau)^{d-\Delta_i}\varphi^i_{(0)}(\boldsymbol{x}) + \ldots + (-\tau)^{\Delta_i}\varphi^i_{(\Delta_i)}(\boldsymbol{x}) + \ldots$$

- Massless and conformal scalars provide useful examples where all time-integrals can be explicitly evaluated in terms of dilogs, *etc*.

Correlators in dS

The most *direct* way to compute dS correlators is via the Schwinger-Keldysh (or in-in) formalism. This features a closed-time path integral:

$$\langle \varphi_{(0)}(\boldsymbol{x}_1)\dots\varphi_{(0)}(\boldsymbol{x}_n)\rangle_{dS} = \int \mathcal{D}\varphi_+(\tau,\boldsymbol{x})\mathcal{D}\varphi_-(\tau,\boldsymbol{x}) \left(\prod_{i=1}^{n}\varphi_{(0)}(\boldsymbol{x}_i)\right) \times \exp\Big(iS_+[\varphi_+] - iS_-[\varphi_-]\Big)$$



There are two distinct fields:

$\varphi_+(\tau,\boldsymbol{x})$ lives on the 'forwards' and $\varphi_-(\tau,\boldsymbol{x})$ on the 'backwards' part of the contour.

Differ by $i\epsilon$ prescription: $\tau \to -\infty(1 \mp i\epsilon)$.

The path integral is restricted to configurations where their late-time values agree:

$$\lim_{\tau\to 0}(-\tau)^{\Delta-d}\varphi_\pm(\tau,\boldsymbol{x}) = \varphi_{(0)}(\boldsymbol{x}).$$

Correlators in dS

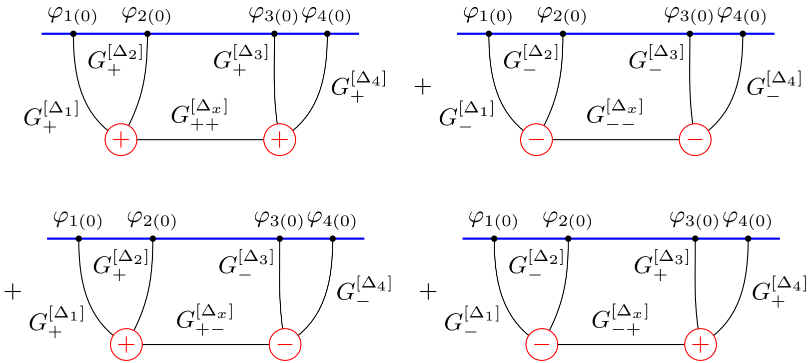
A diagrammatic formalism follows by introducing sources $J_\pm$ for $\varphi_\pm$:

$$Z[J_+, J_-] = \int_{\varphi_+(0,\boldsymbol{x})=\varphi_-(0,\boldsymbol{x})\sim\varphi_{(0)}(\boldsymbol{x})} \mathcal{D}\varphi_+\,\mathcal{D}\varphi_-\,\exp\left[iS_+[\varphi_+]-iS_-[\varphi_-]+i\int \mathrm{d}^{d+1}x\sqrt{-g}(J_+\varphi_+-J_-\varphi_-)\right]$$

- Vertices come in two types $\pm$, according to whether located on the forwards or backwards contour.
- Propagators come in different types according to the vertices they connect.
- Correlators are constructed by summing over diagrams containing all possible assignment of vertex types.

Review: [Chen, Wang, Xianyu 1703.10166]

e.g., for a 4-point exchange diagram



Renormalisation

To renormalise the Schwinger-Keldysh path integral

$$Z[J_+, J_-] = \int_{\varphi_+(0,\boldsymbol{x})=\varphi_-(0,\boldsymbol{x})\sim\varphi_{(0)}(\boldsymbol{x})} \mathcal{D}\varphi_+\,\mathcal{D}\varphi_-\,\exp\Big[iS_+[\varphi_+]-iS_-[\varphi_-]+i\int \mathrm{d}^{d+1}x\sqrt{-g}(J_+\varphi_+-J_-\varphi_-)\Big]$$

it's sufficient to add to the exponent a boundary counterterm of the form

$$iS_{\mathrm{ct}} = i\int_{\tau=0} \mathrm{d}^d\boldsymbol{x}\,(J_+ - J_-)f(\varphi_{(0)};\mu,\mathfrak{a};\epsilon).$$

where f is a *local function* of the late-time field $\varphi_{(0)}(\boldsymbol{x})$ and its derivatives, as well as the RG scale μ and constants $\mathfrak{a}$ parametrising the scheme-dependence.

If multiple bulk fields are present, we add such a counterterm for every field, *i.e.*, $(J_+^k - J_-^k)f^k(\{\varphi_0^l\};\mu,\mathfrak{a}_k;\epsilon)$, where the f^k depend on all late-time fields.

Renormalisation

The local function f^k tells us how the late-time fields are renormalised.

At late times, the Schwinger-Keldysh source terms reduce to

$$\int \mathrm{d}^{d+1}x\sqrt{-g}\,(J_+^k\varphi_+^k - J_-^k\varphi_-^k) \to \int_{\tau=0} \mathrm{d}^d\boldsymbol{x}\,(J_+^k - J_-^k)\varphi_{(0)}^k$$

and combine with the counterterm to become

$$\int_{\tau=0} \mathrm{d}^d\boldsymbol{x}\,(J_+^k - J_-^k)\varphi_{R(0)}^k$$

where $\varphi_{R(0)}^k$ is the *renormalised* late-time bulk field:

$$\varphi_{R(0)}^k = \varphi_{(0)}^k + f^k(\{\varphi_{(0)}^l\};\mu,\mathfrak{a}_k;\epsilon).$$

The form of the f^k is constrained by dimensional analysis:

f^k must have dimension $d-\Delta_k$ to match bulk field $\varphi_{(0)}^k$.

Such counterterms exist *precisely* in the cases for which IR divergences arise.

Renormalisation

In practice, the f^k are polynomials in the late-time fields $\varphi^l_{(0)}$ and their derivatives. For the n-pt function, we need to know this to degree $(n-1)$.

For example, in $d = 3$,

- a massless scalar $(\Delta = 3)$ corresponds to a field $\varphi^{[0]}_{(0)}$ of dimension 0.
- a conformal scalar $(\Delta = 2)$ corresponds to a field $\varphi^{[1]}_{(0)}$ of dimension 1.

The available counterterms are thus:

$$f^{[0]} = a^{[0]}_1 (\varphi^{[0]}_{(0)})^2 + a^{[0]}_2 (\varphi^{[0]}_{(0)})^3 + \ldots, \; f^{[1]} = a^{[1]}_1 \varphi^{[0]}_{(0)} \varphi^{[1]}_{(0)} + a^{[1]}_2 (\varphi^{[0]}_{(0)})^2 \varphi^{[1]}_{(0)} + \ldots$$

The constants $a^{[i]}_j$ are adjusted so that all infinities are cancelled, after which we remove the regulator $\epsilon \to 0$ to obtain the renormalised correlators.

These counterterms are sufficient to renormalise all 24 correlators of massless and conformal scalars at up to 4-points.

Example

For simplicity, let's discuss just a massless scalar with regulated action

$$S_{dS} = -\int \mathrm{d}^{4+2\epsilon}x\sqrt{-g}\Big[\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \frac{1}{2}\epsilon(3+\epsilon)\varphi^2 + \frac{1}{6}\lambda_3\varphi^3 - \frac{1}{24}\lambda_4\varphi^4\Big].$$

We add to the Schwinger-Keldysh exponent the boundary counterterms

$$iS_{\text{ct}} = i\int_{\tau=0} \mathrm{d}^{3+2\epsilon}\boldsymbol{x}\,(J_+ - J_-)\Big[\frac{1}{2}\lambda_3\mathfrak{r}_{[333]}\varphi_{(0)}^2 + \Big(\frac{1}{2}\lambda_3^2\mathfrak{r}_{[33;33x3]} + \frac{1}{6}\lambda_4\mathfrak{r}_{[3333]}\Big)\varphi_{(0)}^3\Big]$$

where $\mathfrak{r}_{[333]}$, $\mathfrak{r}_{[3333]}$ and $\mathfrak{r}_{[33;33x3]}$ are constants to be fixed. At 3 points,

$$\begin{aligned}
&\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\varphi_{(0)}(\boldsymbol{x}_3)\rangle_{\text{ren}} \\
&\quad= \lim_{\epsilon\to 0}\langle:\Big[\varphi_{(0)}(\boldsymbol{x}_1) + \frac{1}{2}\lambda_3\mathfrak{r}_{[333]}\varphi_{(0)}^2(\boldsymbol{x}_1)\Big]::\Big[\varphi_{(0)}(\boldsymbol{x}_2) + \frac{1}{2}\lambda_3\mathfrak{r}_{[333]}\varphi_{(0)}^2(\boldsymbol{x}_2)\Big]: \\
&\qquad\qquad \times :\Big[\varphi_{(0)}(\boldsymbol{x}_3) + \frac{1}{2}\lambda_3\mathfrak{r}_{[333]}\varphi_{(0)}^2(\boldsymbol{x}_3)\Big]:\rangle_{\text{reg}} + O(\lambda_3^2) \\
&\quad= \lim_{\epsilon\to 0}\Big[\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\varphi_{(0)}(\boldsymbol{x}_3)\rangle_{\text{reg}} \\
&\qquad + \lambda_3\mathfrak{r}_{[333]}\left(\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\rangle_{\text{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_3)\rangle_{\text{reg}} + 2\text{ perms}\right)\Big] + O(\lambda_3^2).
\end{aligned}$$

Cancelling pole in reg 3pt fn: $\mathfrak{r}_{[333]} = \frac{1}{3}\Gamma(\epsilon)\mu^{-\epsilon}\big[1 + \epsilon\,\mathfrak{a}_{[333]}^{(1)} + \epsilon^2\mathfrak{a}_{[333]}^{(2)} + O(\epsilon^3)\big]$.

Example

At 4 points,

$$\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\varphi_{(0)}(\boldsymbol{x}_3)\varphi_{(0)}(\boldsymbol{x}_4)\rangle_{\mathsf{ren}}$$
$$= \lim_{\epsilon\to 0}\{\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\varphi_{(0)}(\boldsymbol{x}_3)\varphi_{(0)}(\boldsymbol{x}_4)\rangle_{\mathsf{reg}}$$
$$+ \lambda_3\mathfrak{r}_{[333]}\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\rangle_{\mathsf{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_3)\varphi_{(0)}(\boldsymbol{x}_4)\rangle_{\mathsf{reg}} + [11\text{ perms.}]$$
$$+ \lambda_3^2\mathfrak{r}_{[333]}^2\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\rangle_{\mathsf{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_2)\varphi_{(0)}(\boldsymbol{x}_3)\rangle_{\mathsf{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_3)\varphi_{(0)}(\boldsymbol{x}_4)\rangle_{\mathsf{reg}} + [11]$$
$$+ \ (3\lambda_3^2\mathfrak{r}_{[33;33x3]} + \lambda_4\mathfrak{r}_{[3333]})$$
$$\times\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_2)\rangle_{\mathsf{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_3)\rangle_{\mathsf{reg}}\langle\varphi_{(0)}(\boldsymbol{x}_1)\varphi_{(0)}(\boldsymbol{x}_4)\rangle_{\mathsf{reg}} + [3]\} + O(\lambda_3^3).$$

The regulated 4-pt function (top line of RHS) receives contributions $\sim \lambda_3^2\epsilon^{-2}$ from exchanges and $\sim \lambda_4\epsilon^{-1}$ from contacts. Cancelling divergences fixes:

$$\mathfrak{r}_{[3333]} = -\frac{1}{3}\Gamma(2\epsilon)\mu^{-2\epsilon}\left[1 + \epsilon\,\mathfrak{a}^{(1)}_{[3333]} + O(\epsilon^2)\right],$$
$$\mathfrak{r}_{[33;33x3]} = \frac{1}{18}\Gamma^2(\epsilon)\mu^{-2\epsilon}\left[1 + \epsilon\left(2\mathfrak{a}^{(1)}_{[333]} + \tfrac{1}{3}\right) + \epsilon^2\mathfrak{a}^{(2)}_{[33;33x3]} + O(\epsilon^3)\right].$$

Scheme dep: $\mathfrak{a}^{(1)}_{[333]}$ in 3pt fn; $\mathfrak{a}^{(1)}_{[3333]}$ in 4pt contact; $\mathfrak{a}^{(1,2)}_{[333]}$ and $\mathfrak{a}^{(2)}_{[33;33x3]}$ in exch.

Renormalisation in AdS

Let's now compare this procedure with holographic renormalisation in AdS.

In AdS, for scalars of mass $m_i^2 = \Delta_i(\Delta_i - d)L_{AdS}^{-2}$ such that $d/2 < \Delta_i < d$, we have the near-boundary asymptotics

$$\varphi^i(z, \boldsymbol{x}) = \underbrace{z^{d-\Delta_i}\varphi^i_{(0)}(\boldsymbol{x}) + \dots}_{\text{source}} \quad \underbrace{+\, z^{\Delta_i}\varphi^i_{(\Delta_i)}(\boldsymbol{x}) + \dots}_{\text{operator } \mathcal{O}_i}$$

The renormalisation of tree-level correlators requires two steps:

1. We add local boundary counterterms to the regulated on-shell bulk action:

$$S_{\text{on-shell}}[\varphi^i_{(0)}; \epsilon] \to S_{\text{on-shell}}[\varphi^i_{(0)}; \epsilon] + S_{ct}[\varphi^i_{(0)}; \epsilon]$$

   From the dual CFT perspective, these counterterms involve only the CFT sources and encode the contributions from conformal anomalies.

[de Haro, Solodukhin & Skenderis '00]

Renormalisation in AdS

❷ We renormalise the sources: schematically, up to 3-points, we have

$$\varphi^i_{(0)} = \varphi^i_{(0)}[\phi^j_{(0)}, \epsilon] = \phi^i_{(0)} + \frac{1}{\epsilon}\Box^{k_1}\phi^{j_1}_{(0)}\Box^{k_2}\phi^{j_2}_{(0)} + \dots$$

(This is only possible where the dimension of the 2nd term matches the 1st, but this is precisely the condition for short-distance singularities in the 3-pt fn.)

The renormalised correlators are now obtained by functionally differentiating the renormalised on-shell action,

$$S_{\rm ren}[\phi^i_{(0)}] = \lim_{\epsilon\to 0}\Big[S_{\rm on-shell}[\varphi^i_{(0)}[\phi^j_{(0)}, \epsilon]; \epsilon] + S_{ct}[\varphi^i_{(0)}[\phi^j_{(0)}, \epsilon]; \epsilon]\Big],$$

with respect to these *renormalised* sources $\phi^i_{(0)}$.

Renormalisation in AdS

From the dual CFT perspective, we have the coupling

$$S_{\mathrm{CFT}}[\varphi^i_{(0)}, \mathcal{O}_i] = \int \mathrm{d}^d \boldsymbol{x}\, \varphi^i_{(0)} \mathcal{O}_i.$$

The renormalisation of sources

$$\varphi^i_{(0)} = \varphi^i_{(0)}[\phi^j_{(0)}, \epsilon] = \phi^i_{(0)} + \frac{1}{\epsilon} \Box^{k_1} \phi^{j_1}_{(0)} \Box^{k_2} \phi^{j_2}_{(0)} + \ldots$$

amounts to adding counterterms that renormalise this coupling:

$$S^{ct}_{\mathrm{CFT}}[\phi^i_{(0)}, \mathcal{O}_i; \epsilon] = \frac{1}{\epsilon} \int \mathrm{d}^d \boldsymbol{x}\, \Box^{k_1} \phi^{j_1}_{(0)} \Box^{k_2} \phi^{j_2}_{(0)} \mathcal{O}_i$$

While all β-functions vanish at a critical point, their derivatives wrt sources in general do not, and these counterterms encode this data.

[Bzowski, PM, Skenderis '15]

Renormalisation in AdS vs dS

Comparing the asymptotics,

$$\mathrm{AdS}: \quad \varphi^i(z,\boldsymbol{x}) = z^{d-\Delta_i}\varphi^i_{(0)}(\boldsymbol{x}) + \ldots \quad + z^{\Delta_i}\varphi^i_{(\Delta_i)}(\boldsymbol{x}) + \ldots$$

CFT source — operator $\mathcal{O}_i$

$$\mathrm{dS}: \quad \varphi^i(\tau,\boldsymbol{x}) = (-\tau)^{d-\Delta_i}\varphi^i_{(0)}(\boldsymbol{x}) + \ldots + (-\tau)^{\Delta_i}\varphi^i_{(\Delta_i)}(\boldsymbol{x}) + \ldots$$

late-time field — $\sim$ sources $(J^i_+ - J^i_-)$

- The renormalisation of *sources* in AdS, $\varphi^i_{(0)} = \phi^i_{(0)} + f^i(\{\phi^j_{(0)}\};\epsilon)$ is analogous to that of *late-time fields* in dS, $\varphi^i_{R(0)} = \varphi^i_{(0)} + f^i(\{\varphi^j_{(0)}\};\epsilon)$.

  i.e., AdS counterterms $f^i(\{\phi^j_{(0)}\};\epsilon)\mathcal{O}_i$ vs. $f^i(\{\varphi^j_{(0)}\};\epsilon)(J^i_+ - J^i_-)$ in dS.

- However, in AdS we have the counterterms $S_{ct}[\{\varphi^i_{(0)}\};\epsilon]$ generating anomalies. These have no counterpart in dS.

Renormalisation in AdS vs dS

The reason why can be seen from the Schwinger-Keldysh path integral:

$$Z[J_+, J_-] = \int_{\varphi_+(0,\boldsymbol{x})=\varphi_-(0,\boldsymbol{x})\sim\varphi_{(0)}(\boldsymbol{x})} \mathcal{D}\varphi_+ \, \mathcal{D}\varphi_- \, \exp\Big[iS_+[\varphi_+] - iS_-[\varphi_-] + i\int \mathrm{d}^{d+1}x\sqrt{-g}(J_+\varphi_+ - J_-\varphi_-)\Big]$$

Adding boundary counterterms sends

$$S_+[\varphi_+] \to S_+[\varphi_+] + S_{ct}[\varphi_{(0)}, J_+], \qquad S_-[\varphi_-] \to S_-[\varphi_-] + S_{ct}[\varphi_{(0)}, J_-].$$

Counterterms of the anomaly type $S_{ct}[\varphi_{(0)}]$ are independent of the sources $J_\pm$.

They simply cancel between the forwards and backwards part of the contour, since $Z[J_+, J_-]$ depends only on the *difference* $S_+[\varphi_+] - S_-[\varphi_-]$.

Thus, we can have anomalies in AdS but not in dS. For example, the 2-point function in AdS can have logs, but that in dS is always a pure power law.

See also: [Raju et al, '23]

Lessons for dS/CFT

The existence of a holographic duality between dS and a local CFT requires:

- The *structure* of local IR divergences in dS matches that of local UV divergences in the dual CFT.

This is highly nontrivial since, for specific sets of dimensions $\{\Delta_i, d\}$, CFT correlators have UV divergences associated with anomalies for which there are no corresponding IR divergences in dS.

- One possible resolution is to set up a holographic dictionary based on *analytic continuation* from AdS to dS.

  [Maldacena '02], [PM & Skenderis '09]

  It turns out that anomaly contributions to AdS/CFT correlators are automatically projected out as a result of their ultralocal structure.

Lessons for dS/CFT

Other recent works have explored analytic continuations from dS to AdS theories featuring fields of the shadow dimensions $\bar{\Delta}_i = d - \Delta_i$.

[Sleight & Taronna '20], [di Pietro, Gorbenko & Komatsu '21]

Here, however, cases arise where the dS correlators are IR divergent and require renormalisation while the corresponding shadow CFT correlators are finite.

e.g., 3-point function of two conformal and one massless scalar in dS is

$$\mathrm{ds}^{ren}_{[322]} = \frac{1}{4q_1^3 q_2 q_3}\left\{-q_1 + (q_2+q_3)\left[\log\left(\frac{q_t}{\mu}\right) + \mathfrak{a}_{[322]} - 1\right]\right\}$$

whereas

$$\langle \mathcal{O}_0 \mathcal{O}_1 \mathcal{O}_1 \rangle_{CFT} = c_{[011]} \frac{(q_2+q_3)}{q_1^3 q_2 q_3}$$

i.e., only the the scheme-dependent terms in the dS correlator are reproduced.

Conclusions

IR divergences in de Sitter correlators can be removed by adding local counterterms at future infinity to the Schwinger-Keldysh path integral.

- Only a single type of counterterm is needed for all tree-level correlators.
- The renormalisation of late-time fields in dS corresponds to the renormalisation of sources in AdS/CFT, but anomalies are absent.
- Explicit results available for all renormalised correlators of massless and conformal scalars up to 4-points.
- Open directions: loops, heavy fields, bootstrapping renormalised de Sitter correlators from the *inhomogeneous* conformal Ward identities they obey.