Rigidity and floppiness in field theories and gravity

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Rigidity

- One of the key ambitions of the string theory research programme is to uncover quantum theory of gravity in our universe. But how can we possibly hope to get there in the absence of detailed input from experiment?
- Chapter 1 of Polchinski's 1998 book records a widely expressed hope in the following words: "*We are fortunate that consistency turns out to be such a restrictive principle, since the unification of gravity with the other interactions takes place at such high energy, m_P, that experimental tests will be difficult and indirect.*"
- Polchinski was referring to the rigidity of string S matrix computed perturbatively (as opposed to QFT S matrices computed perturbatively; more details below).
- 27 years later, it is interesting to inquire if fresh evidence has accumulated for this hope.

Rigidity of CFTs: I

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- The one (relatively) uncontroversial example we have of a non perturbative formulation of a quantum theory of gravity is through AdS/CFT. The (or, perhaps, a) non perturbative formulation of a gravitational theory in an asymptotically AdS background is given by the boundary CFT.
- CFTs are fixed points of the renormalization group. Happily they do appear to be (relatively) rigid structures (e.g. this rigidity underlies the universality of critical phenomena).
- This rigidity seems to confirm the expectation spelt out in the remarks in Polchinski's book for 'AdS correlators (see below for more)

Floppiness of QFT S matrices

- In contrast, S matrices in QFTs are inherently floppy structures. For example consider 4 electron scattering. If computed within QED it depends on the mass and charge of the electron. If computed within the standard model, it depends on all the parameters of that theory. If computed within some grand unified or supersymmetric extension of the standard model, it depends on all the parameters of that theory. In choosing the true electron S matrix from all these options we cannot use consistency, but have to rely on some other input like experiment.
- QFT S matrices are, thus, clearly very floppy objects. The space of QFTs itself is floppy, lacking the rigidity of the space of CFTs.

Apparent Paradox and its (obvious) resolution

- We seem to have talked ourselves into a paradox. Let us consider a generic QFT in *AdS* space. We have argued above that this is a floppy space. But don't all these QFTs in *AdS* define a boundary conformal field theory? How is this consistent with the rigidity of the space of CFTs?
- Of course the resolution of this false paradox is obvious. The 'CFTs' constructed by placing quantum field theories in *AdS* spacetimes are pseudo CFTs: they do not have a stress tensor or a local Hamiltonian, and so - even in appropriate gluon like variables - do not obey a 2nd order differential equation in time.
- The only way to get a true CFT on the boundary is to make the bulk gravitational. And, as we saw at the beginning of this talk, it is far from unreasonable to suspect that the space of Gravitational S matrices (i.e. boundary correlators) in *AdS* space is essentially discrete. AdS/CFT asserts that this must be the case.

Interesting Exercise

- It would be interesting to understand this dichotomy between CFTs with and without an stress tensor better from a CFT bootstrap (1).
- The discussion above tells us that while there are many many continuous parameters in the generic solution of the CFT bootstrap equations in CFTs without a stress tensor, we find an essential rigidity with the stress tensor. Why? What precisely is the difference from the point of view of the bootstrap equations? This seems to me to be an important, and potentially tractable, question, whose resolution could lead to important progress in understanding both how gravity is special, from the viewpoint of the S matrix, as well as the CFT bootstrap on its own terms.

Rigidity and the bootstrap I

- Lets make one attempt to understand this rigidity from the viewpoint of the conformal bootstrap. The basic data for a CFT on R^d is its spectrum of operators, i.e. a list (Δ_i, J_i) where i runs over the operators of the theory, and c^a_{ijk} the three point functions between these operators (a here is a multiplicity label that takes into account the fact that higher spin operators have multiple allowed conformally invariant three point structures).

Rigidity and the bootstrap: II

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- The basic bootstrap equation on this data is crossing. Given four operators i, j, k, l, we can compute the correlator from the formula
$$\sum_{m,a,b} c^a_{ijm} c^b_{klm} G^{m,a,b}_{ij:kl}$$
but also from
$$\sum_{n,c,d} c^c_{jkn} c^d_{iln} G^{n,c,d}_{jk:il}$$
where $G^{m,a,b}_{ij:kl}$ is the conformal block representing the fusion of ij to m via the 3 point function c^a_{ijm}, contracted with the fusion of kl to m via the 3 point function c^b_{jkm}.

Rigidity and the bootstrap: III

- Now the blocks $G^{m,a,b}_{ij:kl}$ are both 'basis vectors' in the space of solutions of the Ward identities of conformal invariance (for a 4 point function of ijlm). As a consequence, one can express every block in the second set as a linear combination of blocks in the first set

$$G^{n,c,d}_{jk:il} = \sum_{m,a,b} F(i,j,k,l,m,n|a,b,c,d) G^{m,a,b}_{ij:kl}$$

  The '6 j symbols F are purely kinematical.

- Plugging into the second equation above, and equating coefficients of $G^{m,a,b}_{ij:kl}$, we obtain the equation

$$c^a_{ijm} c^b_{klm} = \sum_{n,c,d} c^c_{jkn} c^d_{iln} F(i,j,k,l,m,n|a,b,c,d) \qquad (1)$$

  Equations labelled by $i,j,k,l,m; a,b$. If we think of operator and degeneracy labels, respectively, as running over N and D values, we have $N^5 D^2$ equations for $\sim N^3 D$ variables.

Gravitational rigidity from the bulk?

- The fact that More eqns than variables in the equation above seems to explain the rigidity of CFTs. As we have seen earlier, however, this cannot be the full explanation, because the counting above is the same for theories with and without a stress tensor.
- As mentioned above would be very very interesting to understand this difference in some detail. Can be worded from the viewpoint of the bulk. Consider an effective field theory in AdS. Why is there such a big difference between such a theory with and without gravity?

Floppiness of Asymptotically Locally AdS Spacetimes

- QFTs are defined by flows (seeded by relevant operators) away from UV CFTs. Thus QFTs can be thought of as floppy deformations of rigid structures.
- Since every atleast some subclasses of CFTs have gravitational dual descriptions (broadly understood), it follows from AdS/CFT that the space of gravitational vacuua can also be floppy. Atleast in the AdS/CFT example we understand that the floppiness has its origins in boundary conditions: while bulk gravitational physics seems rather rigid, boundary conditions can be very floppy.
- From a gravitational point of view, the rigidity of $AdS \times M$ spacetimes is effectively rigidity w.r.t. changing the internal manifold M leaving the AdS part untouched (note that AdS spacetimes do not exist without an internal manifold). Deformations of boundary conditions on the AdS part lead to floppiness.

A question 'asymptotically locally flat spacetimes'

- Everything I have said so far has been about relatively well understood (so safe) world of asymptotically *AdS* spacetimes. In the rest of this presentation, I now make some scattered remarks about asymptotically flat vacuua.
- The first remark (or really question) concerns the existence or otherwise, or non normalizable deformations of flat space. Recall that in the *AdS* context, these deformations were changes in boundary conditions (to asymptotically locally AdS spacetimes) that had a clear interpretation in terms of turning on a local source for the boundary theory. A simple question about classical gravity is the following: do similar deformations exist in asymptotically flat spacetimes? If yes, why have they not been explored more intensively (I have never heard a talk on them). If no, what does this mean about the boundary dual theory (why is it so tight an hard to deform)?

Absolute uniqueness in 11 dimensions?

- Another difference between flat and *AdS* spacetimes is that flat vacuua can exist even without an internal manifold; we believe that this happens in 11 dimensional M theory. It seems very likely that Lorentz invariant gravitational S matrices simply do not exist in higher than 11 dimensions (in the same way that non free CFTs likely do not exist in higher than 6 dimensions), and that the gravitational S matrix 11 dimensions is just absolutely unique (this is not the case for 6d CFTs).
- So our boundary structure appears to take a completely unique form in 10 (or 9, according to taste) dimensions.

Almost unique consistent truncations in the classical limit

- Another difference between flat space and *AdS* is the following. Consider the compactification $R^d \times M^{10-d}$ of type II string theory. The worldsheet CFT for this compactification is a direct sum of the R^d and M^{10-d} CFTs. Consequently the (unnormalized) n point functions of vertex operators that lie entirely in the R^d part of the CFT are proportional to each other (the proportionality constant is the partition function of the M CFT). When the worldsheet is a sphere, the manifold has no moduli. So the proportionality constant is simply a number which can be taken out overall outside. We conclude that all such CFTs admit a universal consistent truncation in the classical limit. This consistent truncation is exact in α'. It does not seem to have any analogues for *AdS* compactifications (no such compactification lies entirely within the universal sector: so the non universality of M CFT seeps into all *AdS*.

- Final very speculative point. There seems to be some correspondence between flat space structures in D dimensions and AdS spacetimes in $D - 4$ dimensions. For instance, while we expect Lorentz invariant flat space gravity not to exist above 11 dimensions, the same may be true for AdS spacetimes above 7 dimensions (this is suggested by examples, but also the folk lore that nontrivial CFTs do not exist above 6 dimensions).
- If this is indeed the case, a similar connection should exist for the boundary structures. Very roughly, the flat space boundary structures (very roughly the connection should be dimensional reduction on very small spheres). If this is the case, it suggests that CFTs have a parent non gravitational structures in higher dimensions. If there was any sense at all in which this was the case, it would, of course, be extremely interesting.

Summary

Let me try to summarize the main points I have tried to make in this presentation.

- Correlators in CFTs (with a local stresss tensor) are rigid structures. S matrices in QFTs are floppy structures.
- Maps to the expectation that 'S matrices' (boundary correlators) for gravity in asymptotically AdS spacetimes are rigid, whereas the same quantities are floppy for QFTs
- Would be very interesting to understand the rigidity of gravity in a bottom up manner from the bulk. And the difference between the two situations from the viewpoint of boundary bootstrap. Difference not clear from variable counting.
- Hits for flat space structures: 11d, totally unique? All dimensions, universal consistent truncation in the classical limit? $D->D-4$ relationship to AdS?