

QUANTUM CLUSTER ALGEBRAS & REPRESENTATIONS OF SHIFTED QUANTUM AFFINE ALGEBRAS

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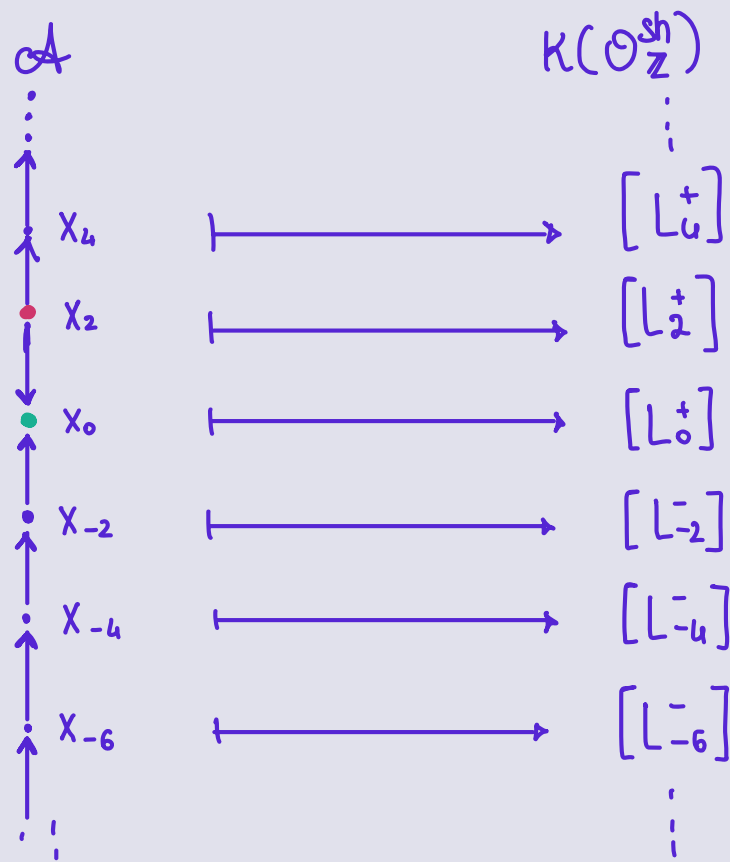
CONTEXT:

- $\mathfrak{g}$ simple, f.d. Lie algebra/ $\mathbb{C}$, type ADE.
- For every μ integral weight of $\mathfrak{g}$, the shifted quantum affine algebra $\mathcal{U}_q^\mu(\hat{\mathfrak{g}})$ is defined (Finkelberg, Tsymbaliuk)
- To study its representation theory, we can look at $\mathcal{O}^\mu$ (Hernandez)
and moreover at $\mathcal{O}^{\text{sh}} := \bigoplus_{\mu} \mathcal{O}^\mu$
- Thanks to a fusion product in $\mathcal{O}^{\text{sh}}$, we can consider the Grothendieck ring $K(\mathcal{O}^{\text{sh}})$.
- There is an injective ring morphism:
$$K(\mathcal{O}^{b, \pm}) \hookrightarrow K(\mathcal{O}^{\text{sh}}).$$

Thm (Geiss-Hernandez-Leclerc)

The Grothendieck ring $K(\mathcal{O}_{\mathbb{Z}}^{\text{sh}})$ has a cluster algebra structure. Namely: there is a cluster algebra $\mathcal{A}$ s.t. $\mathcal{A} \hookrightarrow K(\mathcal{O}_{\mathbb{Z}}^{\text{sh}})$ and $\widehat{\text{Im}}(\mathcal{A}) \cong K(\mathcal{O}_{\mathbb{Z}}^{\text{sh}})$.

example
type A_1



Exchange relation

$$(X_0 X_0' = 1 + X_2 X_{-2}) \longmapsto [L_0^+][L_0^-] = 1 + [L_2^+][L_2^-]$$

QQ-system

Thm (P.)

There exists a quantum cluster algebra structure $\mathcal{A}_t$ for the GHL's cluster algebra.

Let us call $K_t(\mathcal{O}_{\mathbb{Z}}^{\text{sh}})$ the completion of $\mathcal{A}_t$ and we refer to it as the quantum Grothendieck ring of $\mathcal{O}_{\mathbb{Z}}^{\text{sh}}$ (inspired by Varagnolo-Vasserot, Nakajima, Hernandez, Bittmann)

We have $K_t(\mathcal{O}_{\mathbb{Z}}^{b,\pm}) \hookrightarrow K_t(\mathcal{O}_{\mathbb{Z}}^{\text{sh}})$
 $U_q(\hat{\mathfrak{b}})$ -modules

We get quantum QQ-systems as quantum exchange relations.

Thm (P.)

In type A_1 we have the following phenomenon:

- $\mathcal{U}_{t, \text{loc}}^{\pm}(\mathfrak{sl}_2)$ has a quantum cluster algebra structure.
- $\mathcal{U}_{t, \text{loc}}^{\pm}(\mathfrak{sl}_2)$ is isomorphic to $\mathbb{C}_t[SL_2^{w_0, w_0}]$.

COR.

$\mathcal{A}_t$ contains infinitely many copies of $\mathcal{U}_{t, \text{loc}}^{\pm}(\mathfrak{sl}_2)$.

CONCLUSION / QUESTIONS

1. We now have a framework to develop a (q, t) -character theory (as for $U_q(\hat{\mathfrak{g}})$) and hopefully compute many q -characters.
2. We can compare our construction with the one of Qin: are they the same?
3. Is there an analogue result about q -oscillator algebras in other types?

THANK
you !