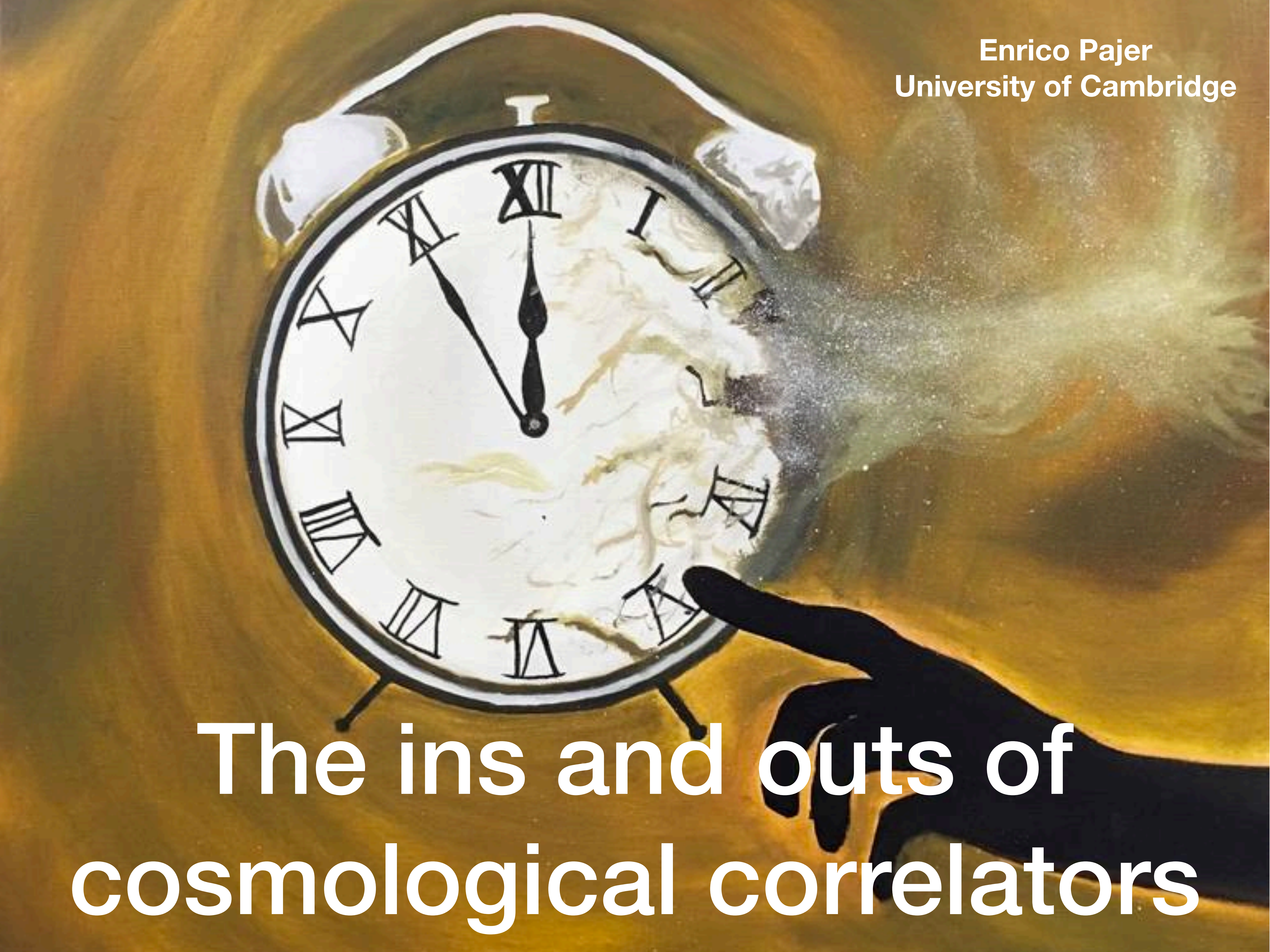

Enrico Pajer
University of Cambridge

The ins and outs of cosmological correlators

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Cosmological observations ~ QFT in curved spacetime

- On large scales (>> Mpc) perturbations evolved almost linearly since the beginning of time.
- Hence cosmological surveys measure QFT correlators of metric fluctuation ζ in curved spacetime

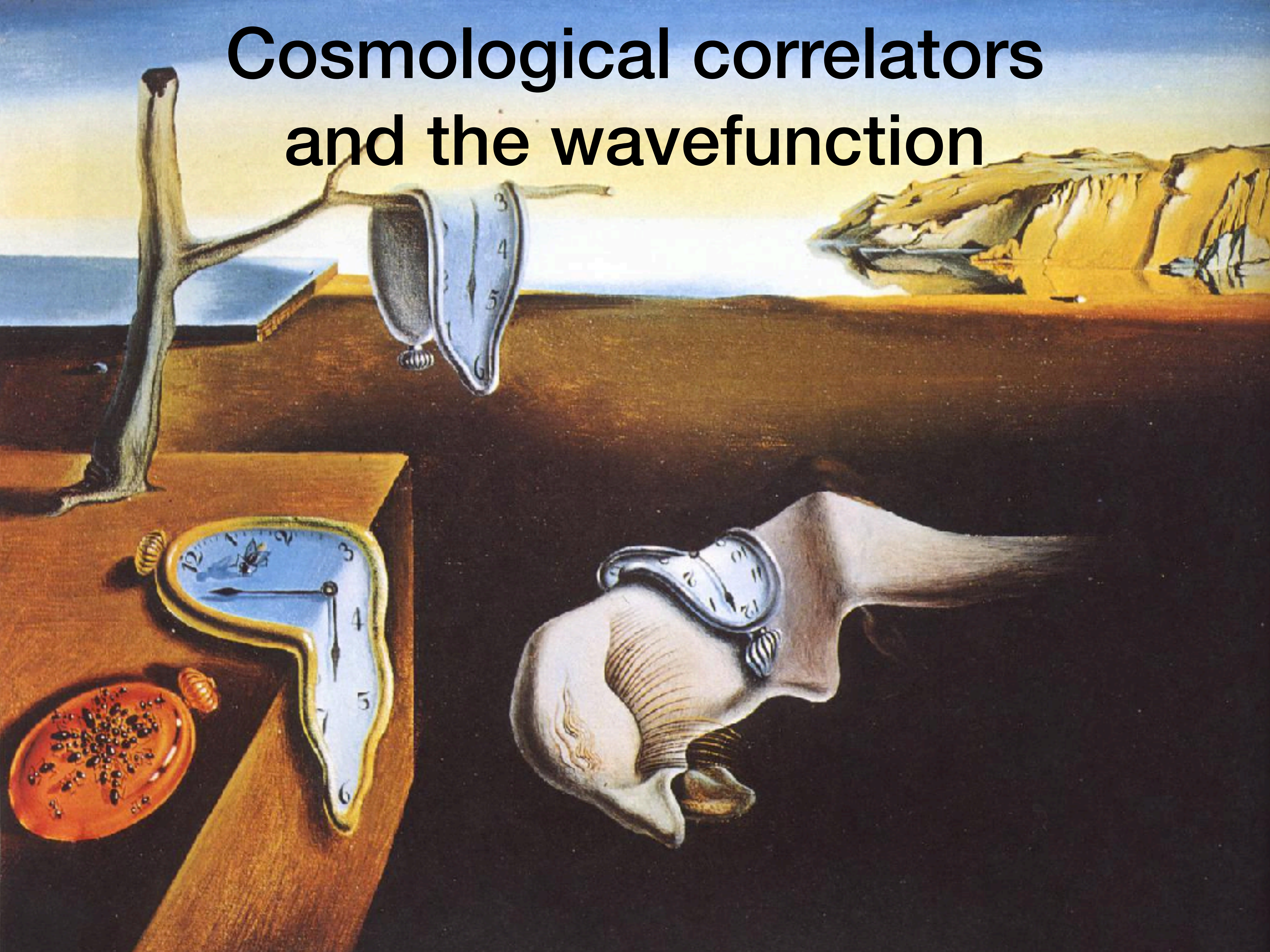
$$\langle \prod_a^n \delta(\mathbf{k}_a) \rangle \sim \left[\prod_b^n \Delta^{(\delta)}(k_b) \right] \langle \prod_a^n \zeta(\mathbf{k}_a) \rangle$$

CMB temperature
density of galaxies
dark matter, …

~

QFT / QG
in de Sitter

- The goal of primordial cosmology is to understand QFT with dynamical gravity (or better Quantum Gravity) in (approximately) de Sitter space


Cosmological correlators
and the wavefunction

de Sitter spacetime

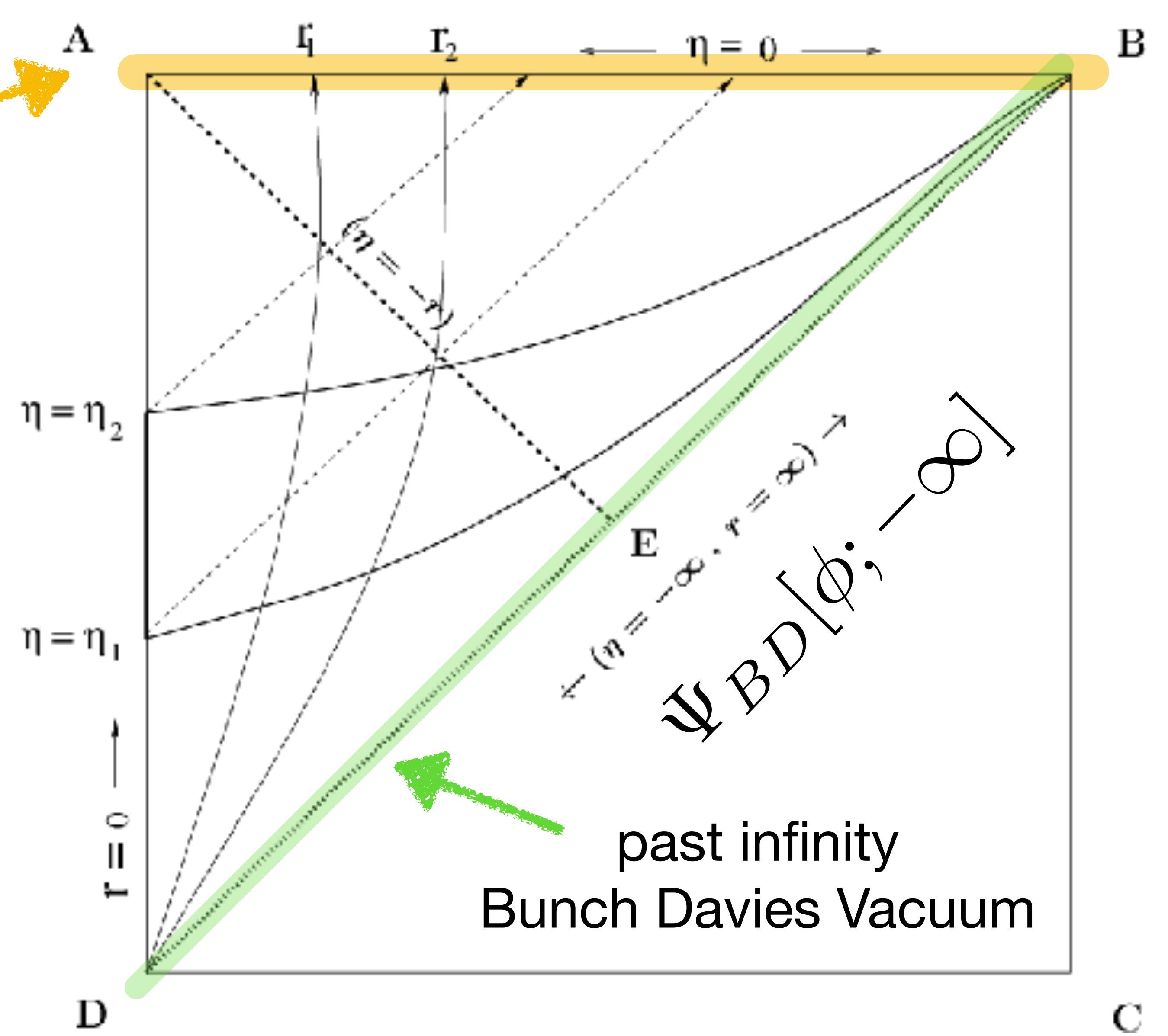
- We work in the Poincare' patch (half of dS),

$$a = e^{Ht} = -1/(H\eta)$$

$$ds^2 = -dt^2 + a^2 dx^2 = a^2(-d\eta^2 + dx^2)$$

$$\Psi[\phi; \eta \to 0]$$

The future (conformal) boundary can be thought as the reheating surface after inflation and determines the statistics of *LSS and CMB observations*



Correlators

- The observables of cosmology are correlators of the product of equal-time local operators $\mathcal{O}$ at $\eta \to 0$

$$\lim_{\eta \to 0} \langle \Omega | \prod_{a=1}^{n} \mathcal{O}(\mathbf{k}_a, \eta) | \Omega \rangle \equiv \langle \prod_{a=1}^{n} \mathcal{O}(\mathbf{k}_a) \rangle \equiv \langle \mathcal{O}^n \rangle \, .$$

- they are usually computed in the "in-in" interaction picture

$$\langle O(\eta) \rangle = \langle 0 \,|\, U^\dagger(-\infty, \eta) O_I(\eta) U(-\infty, \eta) \,|\, 0 \rangle$$

↑ Bunch-Davies initial state ↑ time evolution operator

- They can just as well be computed using the in-out formalism [Donath Pajer 24]

- We know the Feynman rules to compute correlators in perturbation theory

The wavefunction

- The field theoretic *wavefunction* is the projection of the quantum state $|\Psi\rangle$ of the system onto eigenstates $|\phi\rangle$ of the field operators, $\hat{\phi}(x,\eta)|\phi\rangle = \phi(x,\eta)|\phi\rangle$:
$\Psi[\phi,\eta] \equiv \langle\phi|\Psi,\eta\rangle$

- It is a functional of the all fields ϕ in the theory (including the metric) at some time. It can be parameterised in terms of wavefunction coefficients ψ_n

$$\Psi[\phi,\eta] = \exp\left[\sum_{n=2}^{\infty}\int_{\mathbf{k}} \psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n;\eta)\prod_{a}^{n}\phi(\mathbf{k}_a)\right]$$

- This Ψ is a large-volume solution of the wavefunction of the universe, which solves the Wheeler de Witt equation

$$\frac{\text{amplitudes}}{\text{cross sections}} \sim \frac{\text{wavefunction}}{\text{correlators}}$$

- All probabilities can be computed from Ψ as in QM

$$\langle \mathcal{O} \rangle = \int d\phi \Psi^* \mathcal{O} \Psi$$

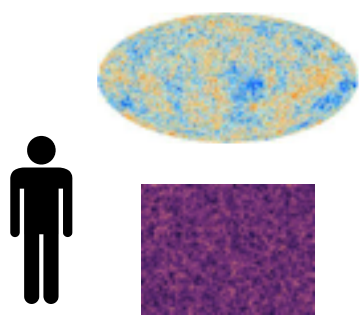
- The Ψ_n are closely related to *cosmological correlators*, which are observables in cosmo data. Note that $B^{\mathrm{PE}}(k) \sim \mathrm{Re}\psi_n$ while $B^{\mathrm{PO}}(k) \sim i\,\mathrm{Im}\psi_n$

$$\langle \phi_{\mathbf{p}}(\eta_0)\phi_{-\mathbf{p}}(\eta_0)\rangle' = \frac{1}{2\,\mathrm{Re}\,\psi_2(p)},$$

$$\left\langle \prod_{a=1}^{3} \phi_{\mathbf{p}_a}(\eta_0) \right\rangle' = -2 \prod_{a=1}^{3} \frac{1}{2\,\mathrm{Re}\,\psi_2'(p_a)} \left[\psi_3(\mathbf{p}) + \psi_3(-\mathbf{p})\right],$$

$$\left\langle \prod_{a=1}^{4} \phi_{\mathbf{p}_a}(\eta_0) \right\rangle' = -2 \prod_{a=1}^{4} \frac{1}{2\,\mathrm{Re}\,\psi_2(p_a)} \Big[\left[\psi_4(\mathbf{p}) + \psi_4(-\mathbf{p})\right] - \frac{\left[\psi_3(\mathbf{p}_1, \mathbf{p}_2, -\mathbf{s}) + \psi_3(-\mathbf{p}_1, -\mathbf{p}_2, -\mathbf{s})\right]\left[\psi_3(\mathbf{p}_3, \mathbf{p}_4, \mathbf{s}) + \psi_3(-\mathbf{p}_3, -\mathbf{p}_4, -\mathbf{s})\right]}{\mathrm{Re}\,\psi_2'(s)} - t - u\Big].$$

The Cosmological Bootstrap



Observed

$\langle\zeta(\mathbf{k})\zeta(\mathbf{k}')\rangle$

Cosmological Bootstrap

Predicted

$\langle\zeta(\mathbf{k})^n\rangle,\ \langle\gamma_{ij}(\mathbf{k})^n\rangle$

Time

Model Building

Unitarity
Locality
Symmetry
...

Perturbative Calculations (in-in,...)

$$\mathcal{L}[\zeta(\mathbf{k},\eta),\gamma_{ij}(\mathbf{k},\eta),\eta]$$

Field theory models

The plan

- I will summarise some constraints on the wavefunction (equivalently on correlators) from:
  - Symmetries
  - Causality: the analytic wavefunction
  - Locality: the manifestly local test (ask me)
  - Unitarity: the cosmological optical theorem
- These vary in generality (some are non-perturbative, for any spacetime, some are only valid for de Sitter) and have been discovered only in the past 4 years!

Broken boost [Green & EP '20]

Assuming only homogeneity, isotropy and scale invariance we have

Theorem: de Sitter symmetries are the largest possible set of symmetries for any single scalar field

Theorem: In single-clock inflation, the only theory of *curvature perturbations* ζ with full de Sitter symmetries is the free theory

Hence the interesting models of inflaton are homogenous, isotropic and scale invariance, but no more (de Sitter boosts are broken)

$$\sum_{a=1}^{n} \vec{k}_a \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{translations}$$

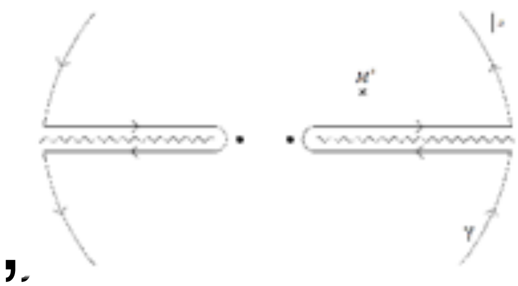
$$\sum_{a=1}^{n} k_a^{[i} \partial_{k_a^{j]}} \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{rotations}$$

$$\sum_{a=1}^{n} (3 - \Delta + k_a \partial_{k_a}) \langle \phi(k_1) \dots \phi(k_n) \rangle = 0 \quad \text{dilations}$$

n

Analyticity and causality

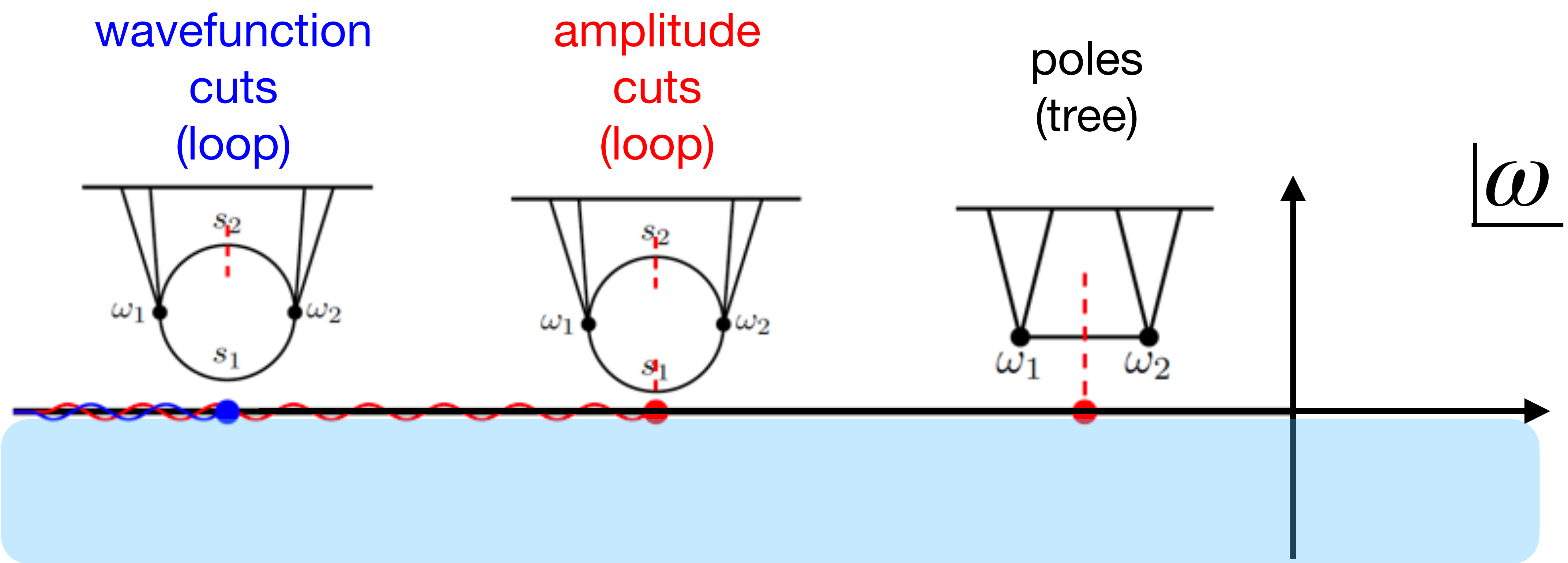
- There is a connection between causality and analyticity, e.g. in complex Mandelstam s for 2-2 scattering
- What is the analytic structure of wavefunction coefficients?



- Results known only for (weaker) "non-relativistic" causality [Arkani-Hamed, Benincasa, Postnikov '17; Benincasa ; Aguei-Salcedo, Lee, Melville & EP '22; Lee '23]
- Define *off-shell wavefunction coefficients* by

$$\psi_n(\{\omega\}, \{\mathbf{k}\}) = \left[\prod_a^n \int d\eta_a K(\omega_a, \eta_a)\right] G^{\text{amp}}(\{\mathbf{k}\})$$

Analyticity



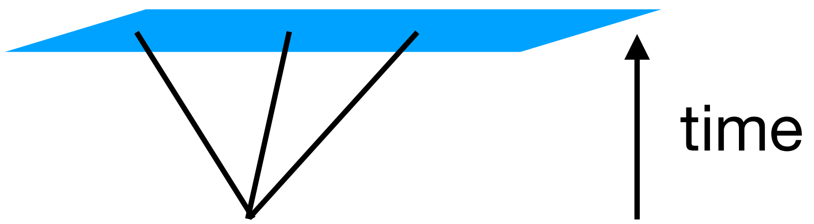
- In Mink, the ψ_n are analytic in ω_a in the lower-half complex plane because the integral is even more convergent. This is a consequence of causality and it is true non-perturbatively
- In pert. theory, there are singularities on the negative when the energy into a vertex vanishes
- Poles at tree-level, cuts at loop level. New “wavefunction” cuts compared to the amplitudes
- Analyticity for massless and conf. coupled field in dS is the same.

Unitarity

- Unitary time evolution, $UU^\dagger = 1$, from a Bunch-Davies state implies infinitely many relations [Goodhew, Jazayeri, EP '20].

- For a contact n-point wavefunction coefficient

$$\psi_n(\{k\}, \{\mathbf{k}\}) + \psi_n^*(-\{k\}, -\{\mathbf{k}\}) = 0$$

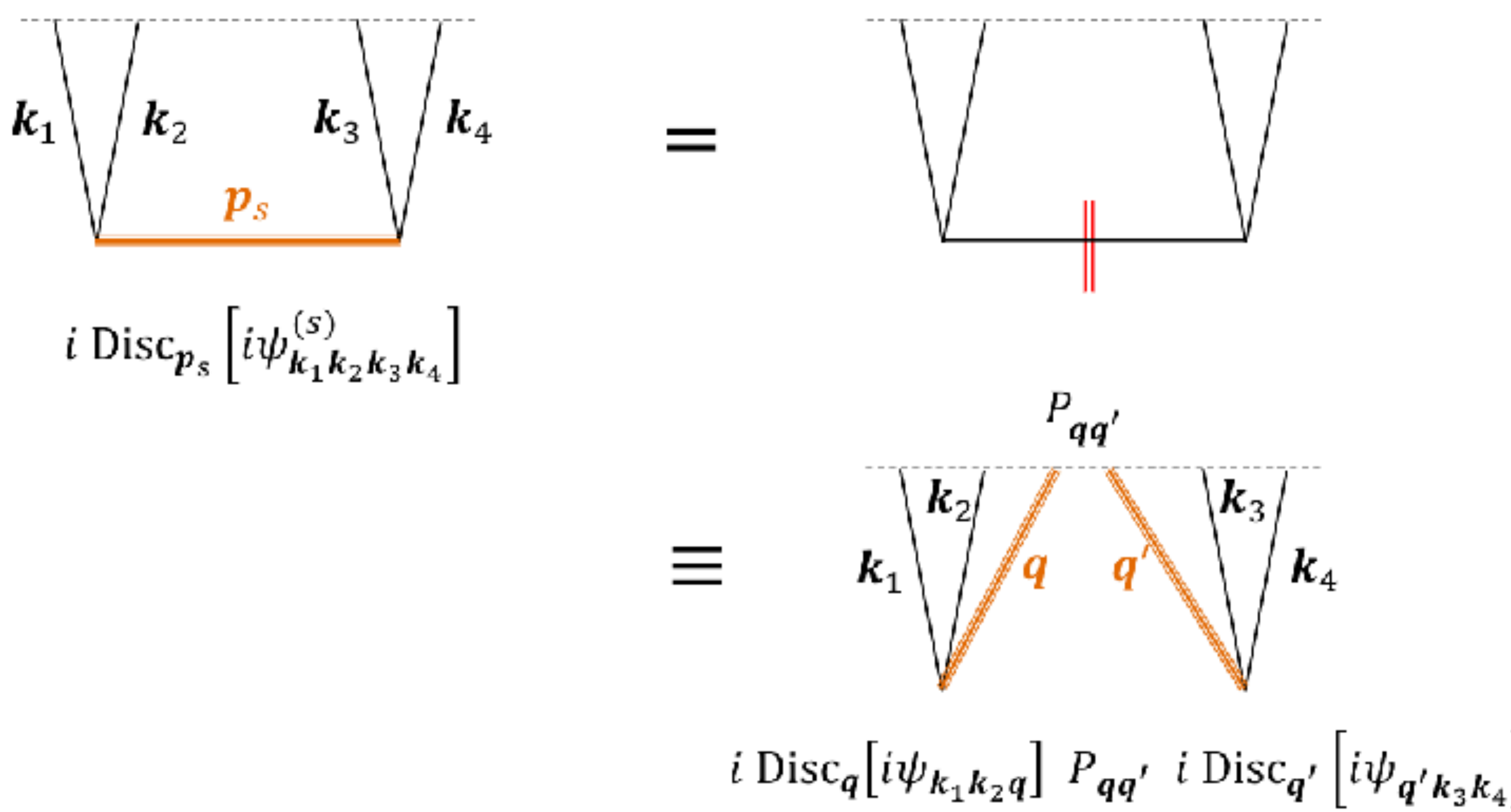


- This is a Cosmological Optical Theorem (COT) and can be interpreted as fixing a "discontinuity"

$$\text{Disc}\ \psi_n \equiv \psi_n(\{k\}, \{\mathbf{k}\}) + \psi_n^*(-\{k\}, -\{\mathbf{k}\}) = 0$$

Exchange diagrams

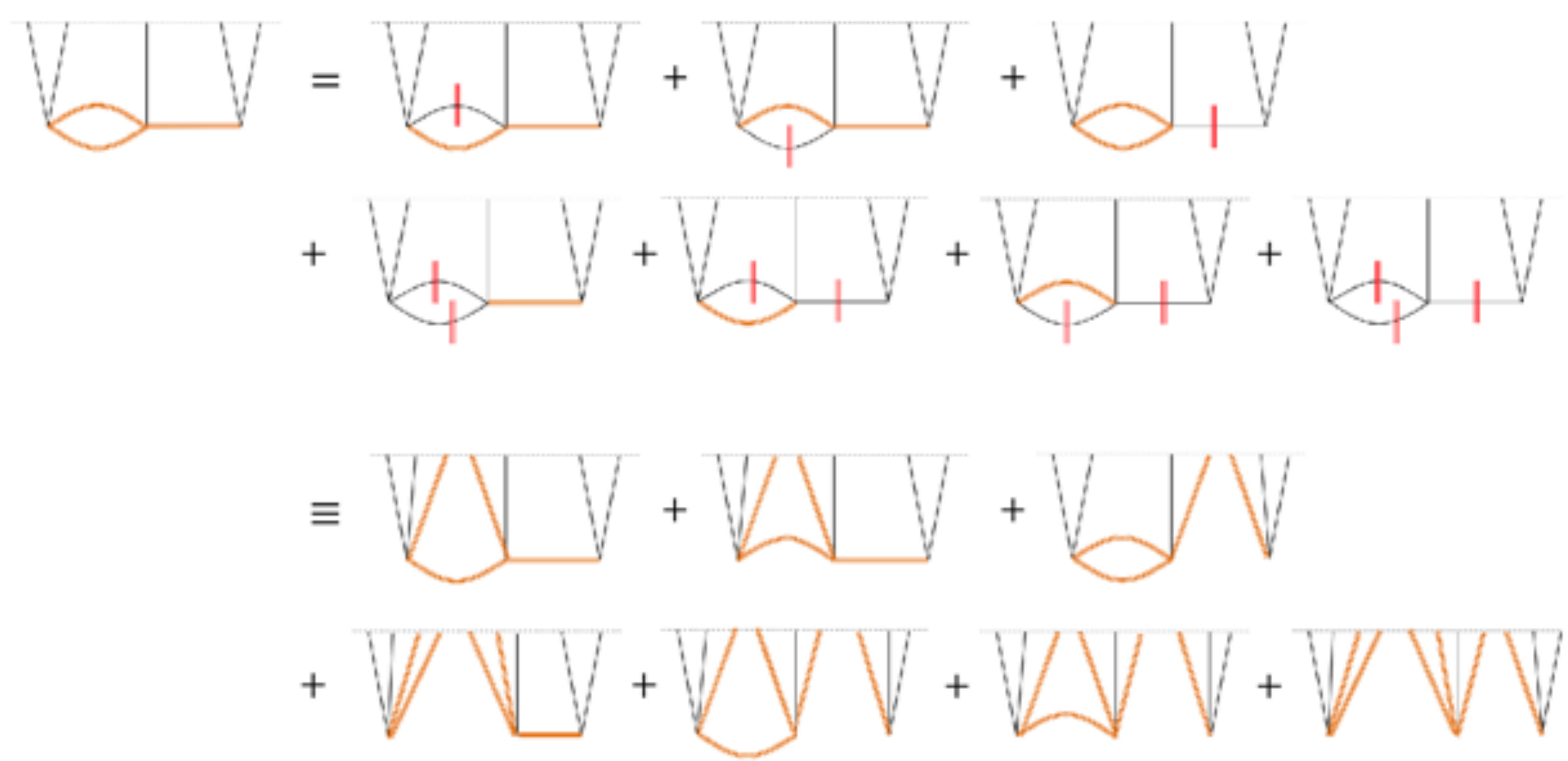
- The next simplest case is a 4-particle exchange diagram (trispectrum). The Cosmo Optical Theorem (COT) is



The Cosmological Optical Theorem (COT)

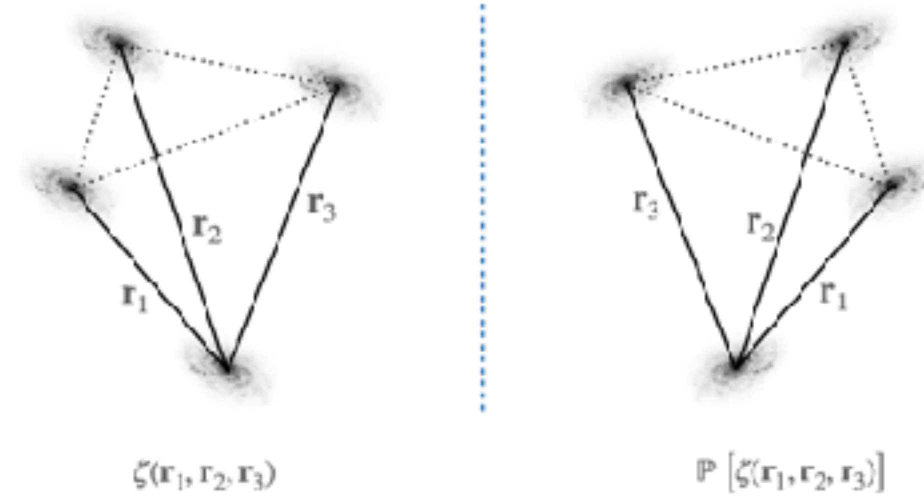
- These relations are valid to *all order in perturbation theory to any number of loops for fields of any mass and spin and arbitrary interactions (around any FLRW admitting a Bunch Davies initial condition)* [Goodhew, Jazayeri & EP '21; Melville & EP '21]

$$i \underset{\substack{\text{internal}\\ \text{lines}}}{\text{disc}} \left[i\,\psi^{(D)} \right] = \sum_{\text{cuts}} \left[\prod_{\substack{\text{cut}\\ \text{momenta}}} \int P \right] \prod_{\text{subdiagrams}} (-i) \underset{\substack{\text{internal \&}\\ \text{cut lines}}}{\text{disc}} \left[i\,\psi^{(\text{subdiagram})} \right] ,$$



Parity-odd correlators, loops and renormalisation

Parity violation



- Laws of nature violate parity. Is there parity violation in our initial conditions?
- The scalar power spectrum (2pt) and bispectrum (3pt) are parity even non-perturbatively (since $\{\mathbf{k}\} \to -\{\mathbf{k}\}$ via $180°$ rotation)
- Parity violation first shows up in the scalar trispectrum B_4 (4pt)

$$\left\langle \prod_{a=1}^{n} \phi(\mathbf{k}_a) \right\rangle = (2\pi)^3 \, \delta_{\mathrm{D}}^{(3)}\left(\sum_{a=1}^{n} \mathbf{k}_a \right) B_n(\{\mathbf{k}\})$$

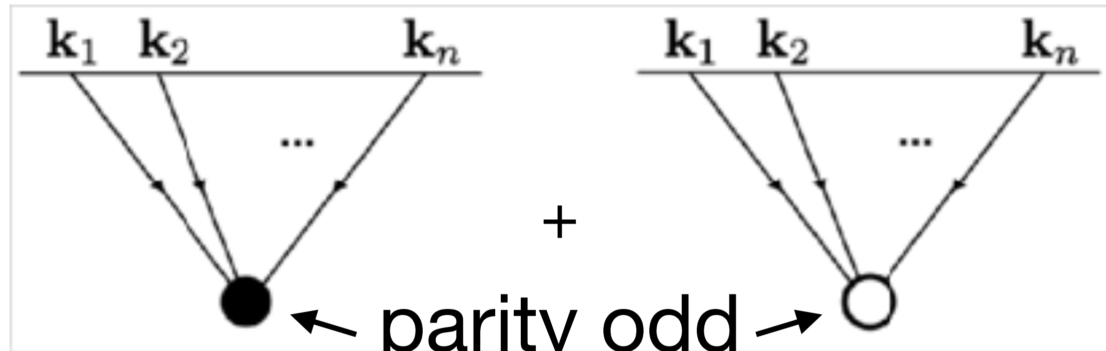
$$B_n^{\mathrm{PO}}(\mathbf{k}_1, \ldots, \mathbf{k}_n) \equiv \frac{1}{2}\left[B_n(\{\mathbf{k}\}) - B_n(-\{\mathbf{k}\})\right]$$

- There are infinitely many parity violating interactions

$$\mathcal{L} \sim \sum_{n_1, n_2, n_3, n_4} \epsilon^{\mu_1 \mu_2 \mu_3 \mu_4} \prod_{a=1}^{4} \partial_{\mu_a} \partial^{n_a} \phi$$

however…

Contact interactions



- Contact interactions contribute to correlators as

$$\langle \varphi(\mathbf{k}_1)\ldots\varphi(\mathbf{k}_n)\rangle = \frac{\int \mathcal{D}\varphi\ \Psi\Psi^*\ \varphi(\mathbf{k}_1)\ldots\varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi\ \Psi\Psi^*}\,,$$

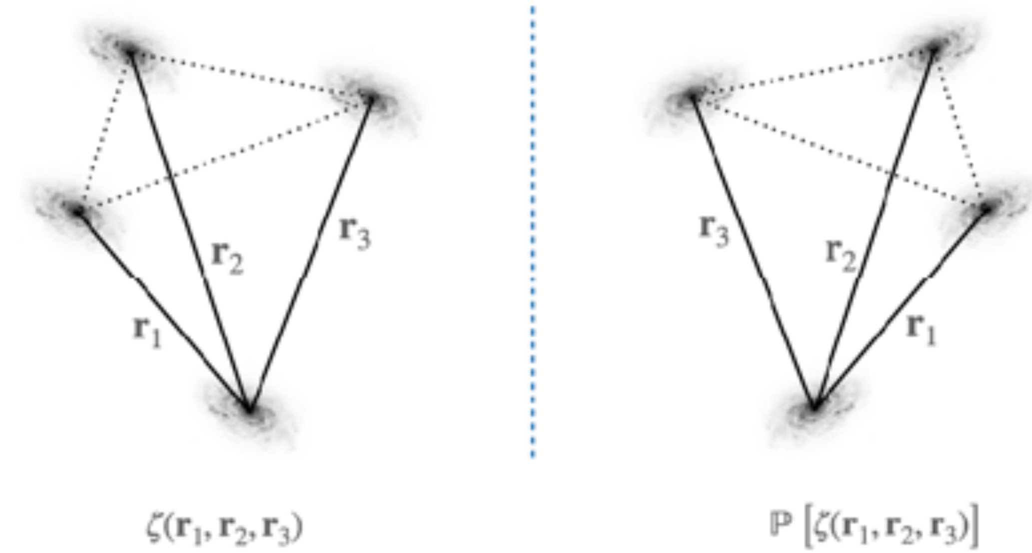
$$B_n^{\rm contact}(\{k\};\{\mathbf{k}\}) = -\frac{\psi_n(\{k\};\{\mathbf{k}\}) + \psi_n^*(\{k\};-\{\mathbf{k}\})}{\prod_{a=1}^n 2\ \mathrm{Re}\ \psi_2'(k_a)}\,,$$

parity odd $\propto \psi_n(\{k\};\{\mathbf{k}\}) - \psi_n^*(\{k\};\{\mathbf{k}\})$

scaling $\propto \psi_n(\{k\};\{\mathbf{k}\}) - (-)^3\psi_n^*(-\{k\};-\{\mathbf{k}\})$

cosmo optical theorem $= 0$

No-go for parity odd

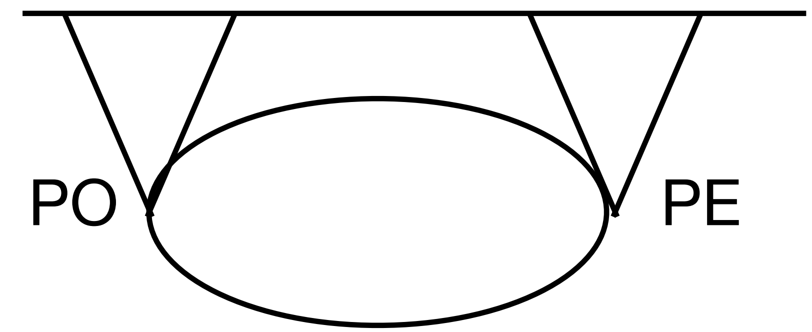


- Assuming scale invariance, *unitarity* and a BD initial state, *IR-finite* parity-odd correlators *vanish* at tree-level for [Liu, Tong, Wang & Xianyu 19; Cabass, Jazayeri, EP & Stefanyszyn '22]
  - Any number of external massless scalars interacting with conformally coupled or massless scalars
  - 4 external massless scalars interacting with any number of massive scalars, or massless fields of any spin
- More generally, the total energy poles of ψ_n^{PO} are always real for intermediate fields of any mass and spin. Hence parity-odd correlators are (sums of) factorised function of different external momenta [Stefanyszyn, Tong & Zhu 23]
- Hence B_4^{PO} is an exceptionally sensitive probe of physics beyond vanilla inflation

Yes-go examples

- There are several yes-go example that relax the above assumptions:
  - exchanging massive spinning fields
  - break scale invariance
  - exchange parity-odd massless spinning fields
  - modified dispersions relations (non Bunch-Davies)
  - *Beyond tree level*

Leading loops



- Because tree-level vanishes, the leading contribution to B_4^{PO} is 1-loop!

- 1-loop 1-vertex vanished in dim reg (no momentum flow)

- We perform the calculation in mass and dimensional regularisation (*m&d reg*), a variant of dim reg where mode functions remain simple in $d \neq 3$. The result is shockingly simple

  [Lee, McCulloch, EP 23]

$$B_4^{\mathrm{PO}} = i \frac{(\mathbf{k}_1 \times \mathbf{k}_2) \cdot \mathbf{k}_3 \, \mathrm{Poly}_p(\mathbf{k})}{(k_1 k_2 k_3 k_4)^3 k_T^p}$$

- This an observable quantum effect and is related to a surprising universal result.

Regulators

- Parity-odd correlators are proportional to the imaginary part of wavefunction coefficients: $B^{\text{PO}}(k) \sim i\,\text{Im}\,\psi$, which arises first at one loop
- Many regulators (others too: Mellin-Barnes, etc):
  - dim reg in dS is cumbersome because mode functions are $\propto H^{(}1)_{\nu(d)}$
  - m&d reg is simpler, but breaks the shift symmetry of a massless scalar
  - We introduce η-regulators in dS [Jain, Pajer Tong 25], adapting from Mink [Padilla & Smith 24]. Simpler calculation, manifest shift symmetry and scale invariance

$$\psi_2 \sim \int_k^\infty \frac{dq_+}{q_+} \frac{\text{Poly}_{m+n}(q/M)}{\text{Poly}_m(q/M)} \eta\left(\frac{q_+/k}{\Lambda/H}\right)$$

  - If (i) $\eta(0) = 1$ and (ii) $\lim_{|x|\to\infty} \eta(x) = 0$ then η-regulators are unitarity and analytic

Regulator independence

- We studied Im ψ_2 in a $\lambda\dot{\phi}^3$ theory. We proved ψ_2 is regulator independent (dim reg, m&d reg, unitary η-reg)

$$\psi_2^{\text{tree}} + \widehat{\psi}_2^{1L} = \frac{1}{H^2}\left(\frac{ik^2}{\eta_0} - k^3\right) + \frac{1}{15}\frac{\lambda^2 H^2}{(4\pi)^2}k^3\left(\frac{i\pi}{2} + \log\frac{\mu}{H}\right)$$

- Only the final renormalised result is regulator independent

Regularisation schemes	Scale inv.	Im. part	Time-indep.	Diff inv.
Dim reg	✗ → ✓	$0 \to +\pi/2$	✓	✓
m&d reg	✓	$+\pi/2$	✗ → ✓	✓
η reg (unitary & analytic)	✓	$+\pi/2$	✓	✓
η reg (real)	✓	0	✓	✓
η reg (complex)	✓	Arbitrary	✓	✓
Partial dim reg (scheme A)	✗	0	✓	✗
Partial dim reg (scheme B)	✗ → ✓	$0 \to +\pi/2$	✓	✗

Universality of Imψ

- All one-loop wavefunction coefficients ψ_n^{1L} in this theory, for any n, obey

$$\widehat{\psi}_n^{\,1\mathrm{L}}(k) = f_0(k)\left(\frac{i\pi}{2} + \log\frac{\mu}{H} + \mathbb{R}\right)$$

- where μ is the renormalisation scale or equivalently the logarithmic UV divergence

- This suggest a deeper relation to the renormalisation group

$$\left(\mu\frac{\partial}{\partial\mu} - \frac{2}{\pi}\,\mathrm{Im}\right)\widehat{\psi}_n^{\,1\mathrm{L}} = 0$$

- We suspect that Im $\psi_n \neq 0$ is related to the scale anomaly of the (bulk) QFT in de Sitter (work in progress). This is cool because the anomaly would be directly related to the observable parity-odd correlators

Horizons

- In the past few years we have understood many general consequences of locality, unitarity and causality for cosmological correlators
- This lead to many predictions, old and new, that are being searched for in cosmological datasets
- Parity-odd correlators vanish at tree-level in the simplest models of inflation, so loop contributions are leading!
- We studied $B^{\text{PO}} \sim \text{Im}\,\psi_n$ by adapting (unitarity and analytic) η-regulators to de Sitter and proving a remarkable universality for all 1-loop contributions.