Computing multiple solutions of systems of nonlinear equations with deflation

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Many systems of nonlinear equations admit multiple solutions, which are often crucial for understanding the problem at hand. Algorithms for computing multiple solutions have been developed over several decades in the context of numerical bifurcation analysis, with great success. However, the core algorithms of continuation, bifurcation detection, and branch switching can miss branches of solutions not connected to the given initial solution. They can also be complex to apply to large-scale discretisations of the nonlinear partial differential equations that arise in many real-world problems. A complementary alternative is to employ deflation, which modifies the nonlinear problem so that the Newton--Kantorovich iteration cannot rediscover known solutions, and thus if the iteration converges it finds a new solution. Deflation enables the discovery of disconnected branches. Moreover, when combined with natural or tangent continuation, the resulting subproblems can be efficiently solved if a good preconditioner is available for the original Newton step, making the method convenient and practical to apply at large scale. In this talk we introduce deflation, discuss how it may be combined with classical approaches to yield effective algorithms for discovering multiple solutions of par2 (2024