The condensed homotopy type of a scheme

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Let X be a locally topologically noetherian scheme. In their paper on the proétale topology, Bhatt and Scholze defined the proétale fundamental group $\pi_1^{\text{proét}}(X)$. The profinite completion of $\pi_1^{\text{proét}}(X)$ recovers the usual étale fundamental group. Moreover, $\pi_1^{\text{proét}}(X)$ agrees with $\pi_1^{\text{ét}}(X)$ when X is normal, but $\pi_1^{\text{proét}}(X)$ has the better property that it classifies $\mathbf{Q}_p$-local systems. In this talk, we'll explain how to use condensed mathematics to define a "condensed homotopy type" whose fundamental group refines the proétale fundamental group. We'll also explain a number of computations of and foundational results about the condensed homotopy type. The ideas from this project originate in our work with Barwick and Glasman, and the paper we'll discuss is joint work with Holzschuh, Lara, Mair, Martini, and Wolf.s