Infinite towers of 2d symmetry algebras from Carrollian limit of 3d CFT

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AdS/CFT meets Carrollian and Celestial Holgraphy, Sept. 2025

based on 2508.19981 w. de Gioia

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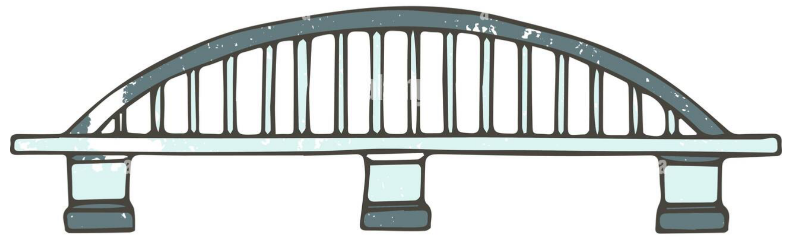
Motivation

AdS/CFT



AFS/CCFT

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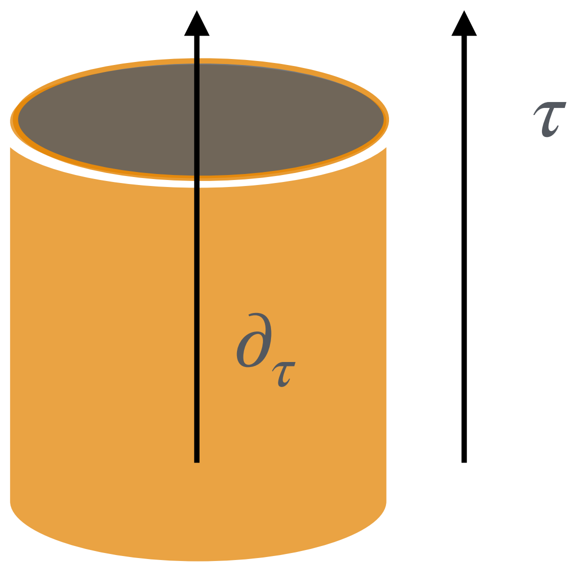

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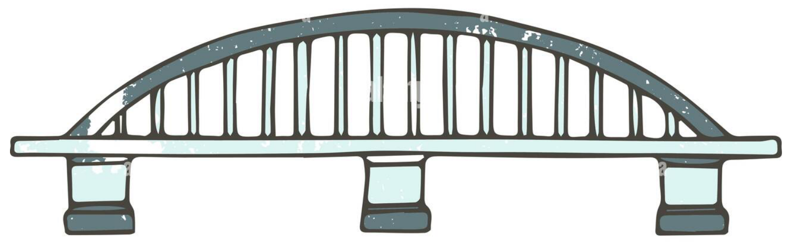


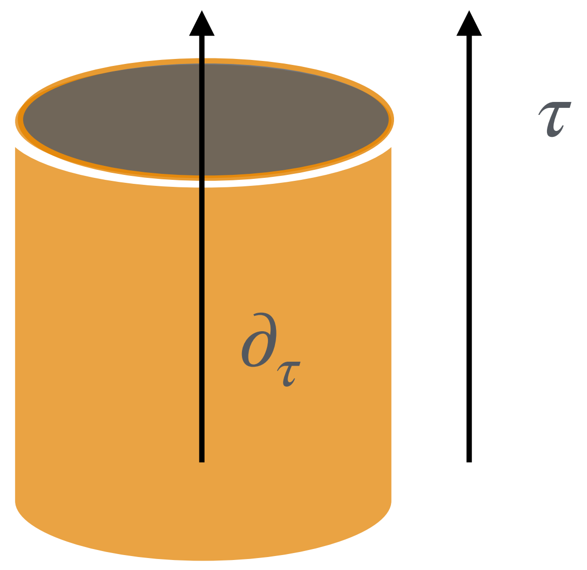
AFS/CCFT

AdS_{d+1} isometries ~ so(d,2)

Bulk time evolution ~ boundary time evolution $\Longrightarrow$

Plausible to postulate that:
- bulk $\mathscr{H}$ ~ boundary $\mathscr{H}$
- bulk unitarity ~ boundary unitarity

Motivation



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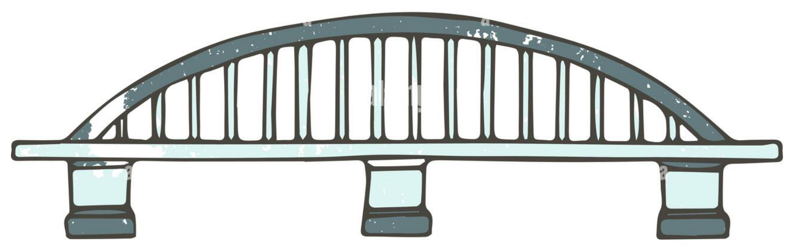

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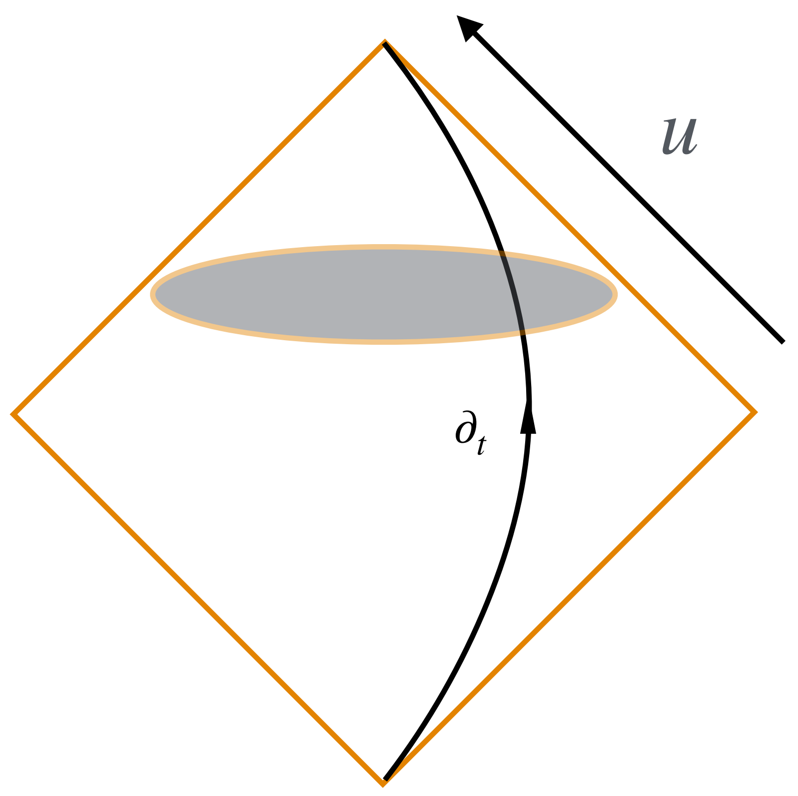

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Infinite-dimensional enhancement of

Poincare symmetry ~ symmetries of Carr./Cel. CFT

Bulk unitarity ~ boundary ??

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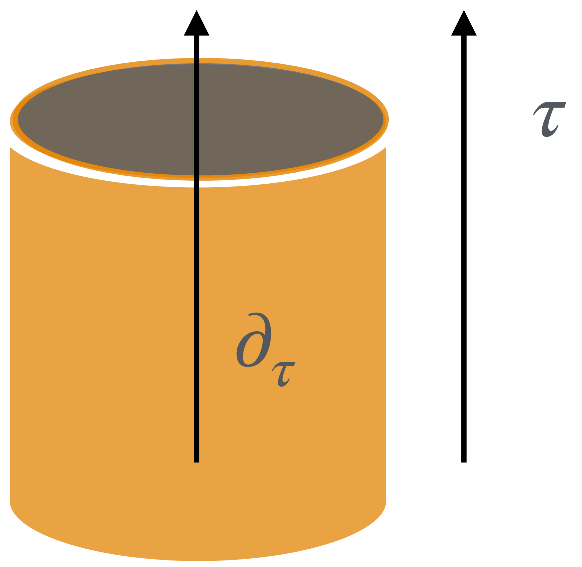

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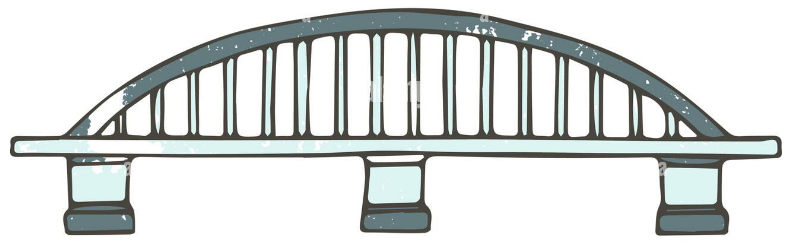

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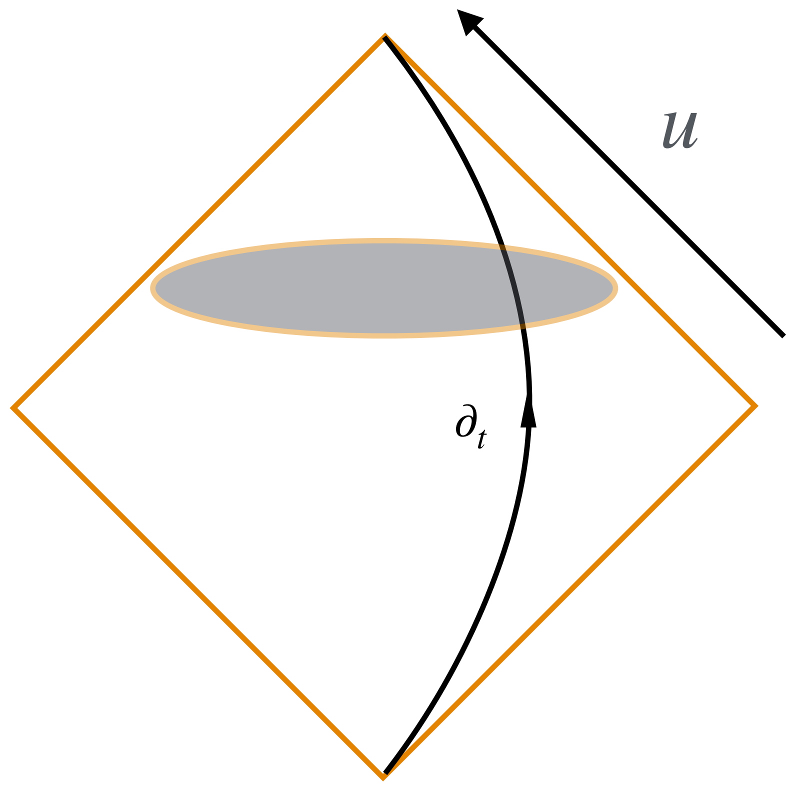

AdS_{d+1} isometries ~ so(d,2) [finite-dimensional symmetries]	Infinite-dimensional enhancement of Poincare symmetry ~ symmetries of Carr./Cel. CFT

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Bulk unitarity ~ boundary ??

Goal

➡ = Bulk-point/flat space/Carrollian limit $\overset{\boxed{d=3}}{\Longrightarrow}$

- Towers of (conformally) soft theorems
- w-infinity and s algebras of CCFT

AdS_{d+1} isometries ~ so(d,2) [finite-dimensional symmetries] ➡ Infinite-dimensional enhancement of Poincare symmetry ~ symmetries of Carr./Cel. CFT

Review: Infinite symmetry algebras in CCFT

[Guevara, Himwich, Pate, Strominger '21; Strominger '21]

Lorentz sl(2,C) → sl(2,R)$_L$ x sl(2,R)$_R$

Conformal primary gluon OPE: $\mathcal{O}^a_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^b_{\Delta_2,+1}(z_2,\bar{z}_2) \sim -\frac{if^{ab}{}_c}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}^c_{\Delta_1+\Delta_2-1,+1}(z_2,\bar{z}_2) + \ldots$

including $sl(2,\mathbb{R})_R$ descendants

$$\sim if^{ab}{}_c\frac{1}{z_{12}}\sum_{n=0}^{\infty}B(\Delta_1-1+n,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}^n\mathcal{O}^c_{\Delta_1+\Delta_1-1}(z_j,\bar{z}_j) + O(z_{ij}^0)$$

computed from expansion of 2d OPE block: $\mathcal{O}^a_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^b_{\Delta_2,+1}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}}\int_0^1 dt\frac{\mathcal{O}^c_{\Delta_1+\Delta_2-1,+1}(z_2,\bar{z}_2+t\bar{z}_{12})}{t^{2-\Delta_1}(1-t)^{2-\Delta_2}}$

Tower of conformally soft gluon theorems: $R^a_s(z_1,\bar{z}_1)\mathcal{O}^b_{\Delta_2}(z_2,\bar{z}_2) \sim if^{ab}{}_c\frac{1}{z_{12}}\sum_{n=0}^{s}\binom{s+1-\Delta_2-n}{s-n}\frac{\bar{z}_{12}^n}{n!}\bar{\partial}^n\mathcal{O}_{\Delta_2-s}(z_2,\bar{z}_2) + O(z_{12}^0)$

residue of conformal primary gluon at $\Delta_i = 1-s,\ s\geq 0$

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including $sl(2,\mathbb{R})_R$ descendants

$$\sim if^{ab}{}_c\frac{1}{z_{12}}\sum_{n=0}^{\infty}B(\Delta_1-1+n,\Delta_2-1)\frac{\bar{z}_{12}^n}{n!}\bar{\partial}^n\mathcal{O}^c_{\Delta_1+\Delta_1-1}(z_j,\bar{z}_j) + O(z_{ij}^0)$$

computed from expansion of 2d OPE block:

$$\mathcal{O}^a_{\Delta_1,+1}(z_1,\bar{z}_1)\mathcal{O}^b_{\Delta_2,+1}(z_2,\bar{z}_2) \sim \frac{-if^{ab}{}_c}{z_{12}}\int_0^1 dt\frac{\mathcal{O}^c_{\Delta_1+\Delta_2-1,+1}(z_2,\bar{z}_2+t\bar{z}_{12})}{t^{2-\Delta_1}(1-t)^{2-\Delta_2}}$$

S-algebra from (light transform of):

$$R^a_s(z_1,\bar{z}_1)R^b_{s'}(z_2,\bar{z}_2) \sim if^{ab}{}_c\frac{1}{z_{12}}\sum_{n=0}^{s}\binom{s+s'-n}{s'}\frac{\bar{z}_{12}^n}{n!}\bar{\partial}^n R^c_{s+s'}(z_2,\bar{z}_2) + O(z_{12}^0)$$

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[Guevara, Himwich, Pate, Strominger '21; Strominger '21]

Lorentz sl(2,C) → sl(2,R)$_L$ x sl(2,R)$_R$

Conformal primary graviton OPE: $O_{\Delta_1,2}(z_1,\bar z_1)O_{\Delta_2,2}(z_2,\bar z_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{\infty} B(\Delta_1-1+n,\Delta_2-1)\frac{\bar z_{12}^{n+1}}{n!}\bar\partial^n O_{\Delta_1+\Delta_2,2}(z_2,\bar z_2)+\mathcal{O}(z_{12}^0)$

computed from analogous expansion of 2d OPE block $O_{\Delta_1,2}(z_1,\bar z_1)O_{\Delta_2,2}(z_2,\bar z_2) \sim \frac{\kappa}{2}\frac{\bar z_{12}}{z_{12}}\int_0^1 dt\frac{1}{(1-t)^{2-\Delta_2}t^{2-\Delta_1}}\mathcal{O}^+_{\Delta_1+\Delta_2}(z_2,\bar z_2+t\bar z_{12})$

Tower of conformally soft graviton theorems:

$$N_s^+(z_1,\bar z_1)O_{\Delta_2,2}(z_2,\bar z_2) \sim -\frac{\kappa}{2}\frac{1}{z_{12}}\sum_{n=0}^{s}\frac{(-1)^{n-s}}{(\Delta_2-1)B(1+s-n,\Delta_2+n-1-s)}\frac{\bar z_{12}^{n+1}}{n!}\bar\partial^n O_{\Delta_1+\Delta_2,2}(z_2,\bar z_2)+\mathcal{O}(z_{12}^0)$$

→ residue of conformal primary graviton at $\Delta_i = 1-s$

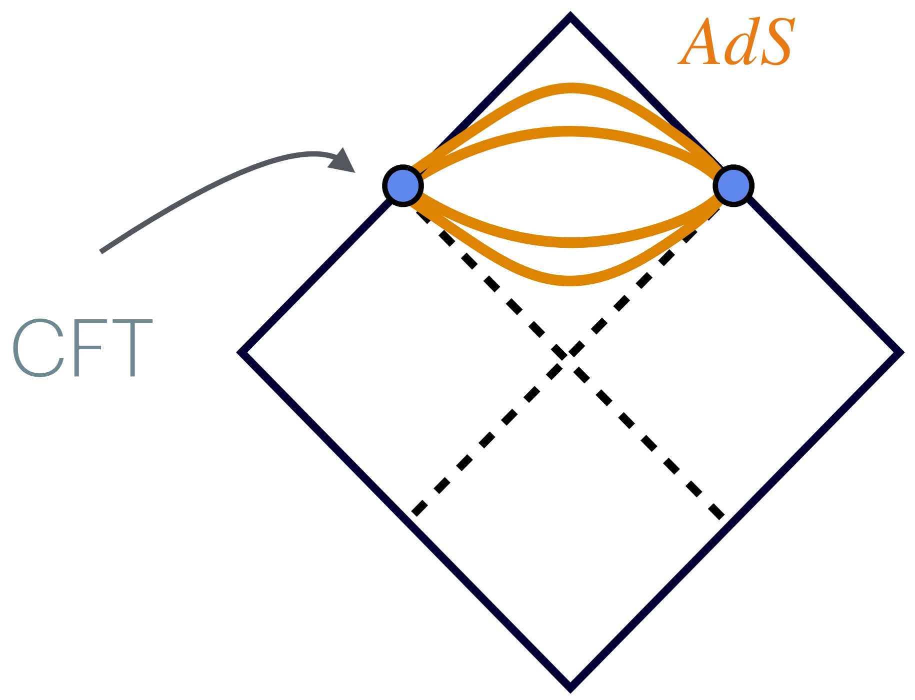
w-algebra from (light transform of) $N_s^+(z_1,\bar z_1)N_{s'}^+(z_2,\bar z_2)$ OPE

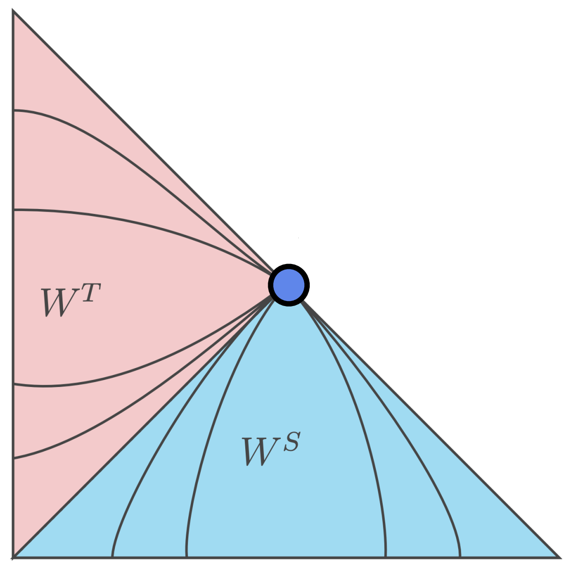
Setup

Embedding space $\mathbb{R}^{3,2}$: 3d conformal algebra = Lorentz transformations

AdS$_4$ bulk: $-(X^0)^2 - (X^4)^2 + \sum_{i=1}^{3} (X^i)^2 = -\ell^2$; CFT$_3$ boundary: $P^2 = 0$

Fields are homogenous ``functions'' of degree Δ: $\Phi(\lambda P) = \lambda^{-\Delta}\Phi(P)$





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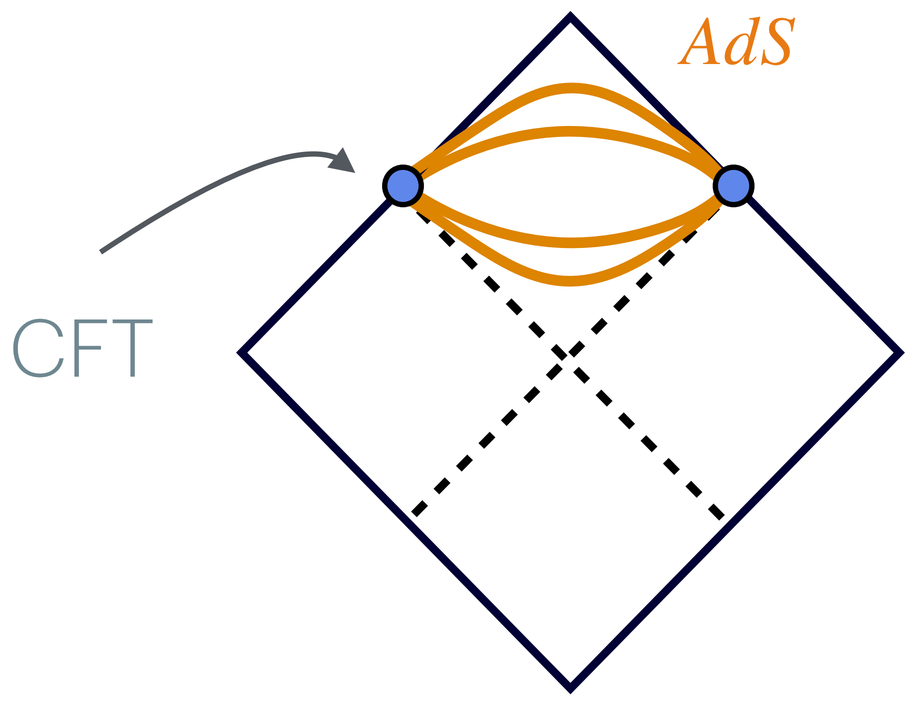

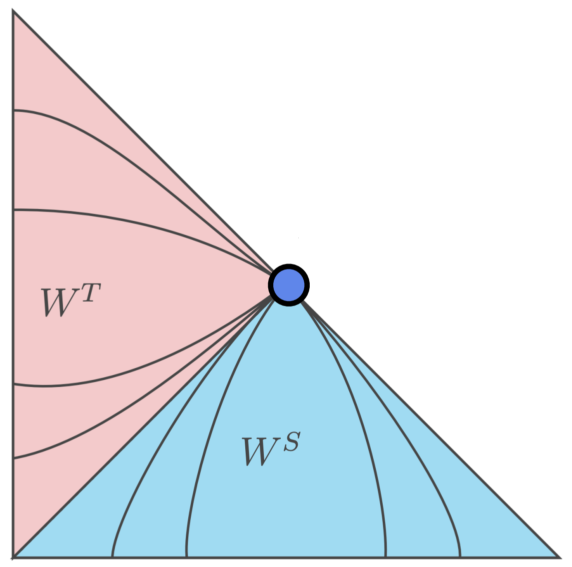

Starting point: OPE block of scalar primaries in 3d CFT:

$$O_{\Delta_1}(P_1)O_{\Delta_2}(P_2)=\sum_{\Delta_3}\mathcal{N}_S\int d^3P_3\langle O_{\Delta_1}(P_1)O_{\Delta_2}(P_2)\underbrace{O^S_{3-\Delta_3}(P_3)}_{\text{shadow}}\rangle O_{\Delta_3}(P_3)+\underbrace{\cdots}_{\text{spinning exchanges}}$$

Shadow primary: $$O^S_{3-\Delta_3}(P_3)\equiv\int d^3P\frac{1}{(-2P_3\cdot P)^{3-\Delta_3}}O_{\Delta_3}(P)$$

Setup

Embedding space $\mathbb{R}^{3,2}$: 3d conformal algebra = Lorentz transformations

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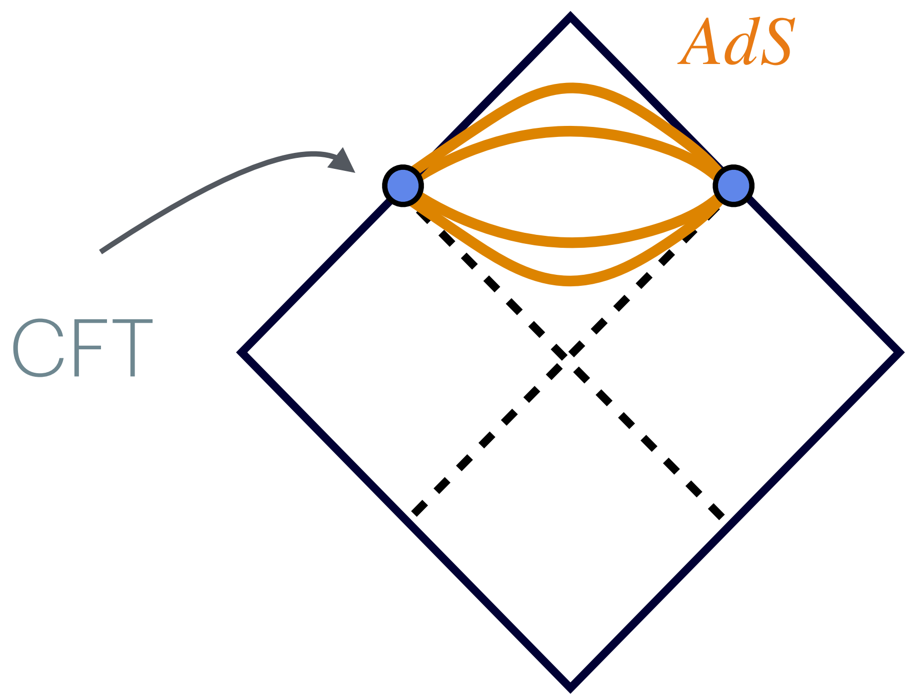

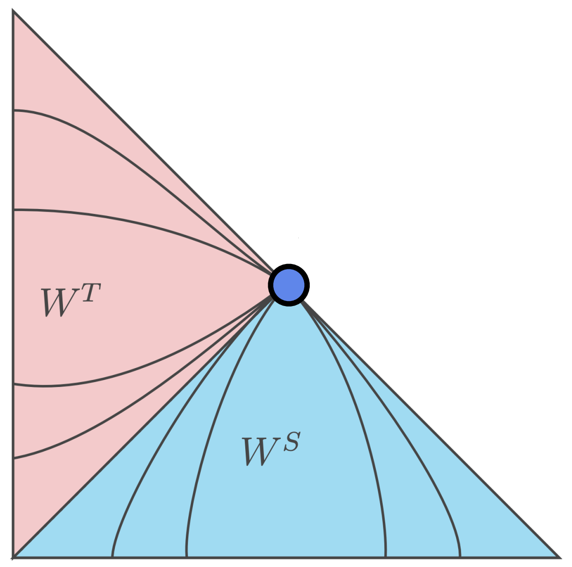

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Scalar three-point function: $\langle O_{\Delta_1}(P_1)O_{\Delta_2}(P_2)O_{\Delta_3}(P_3)\rangle = \dfrac{c_{123}}{(-P_1\cdot P_2)^{\beta_{12}}(-P_2\cdot P_3)^{\beta_{23}}(-P_3\cdot P_1)^{\beta_{31}}}$

$\beta_{12} = \dfrac{\Delta_1 + \Delta_2 - \Delta_3}{2}$

Strategy

AdS/CFT $\Longrightarrow$ gluons dual to conserved currents, gravitons dual to the stress tensor $\Longrightarrow$

- consider OPE blocks of (eg.) the stress tensor in CFT_3

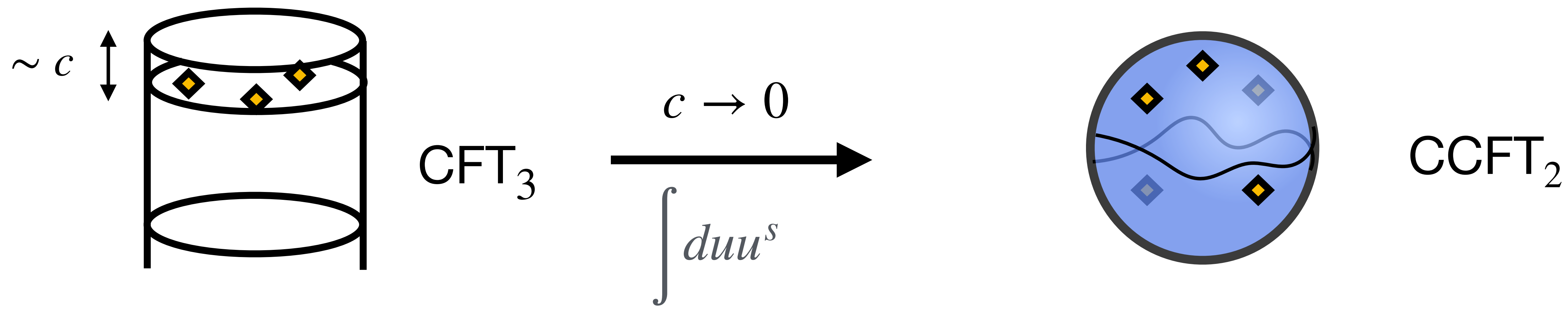
$$T^+(x_1)T^+(x_2) \supset c_{T^+T^+\widetilde{T}^-}\int d^3x_3 \langle T^+(x_1)T^+(x_2)\widetilde{T}^-(x_3)\rangle T^+(x_3) + \cdots \qquad T^+ \equiv Z_+^{\mu\nu}T_{\mu\nu} \leftrightarrow \text{positive helicity graviton}$$

- in the Carrollian limit expect: $\lim_{c\to 0} T^+(cu,z,\bar{z}) \sim \mathcal{T}^+(u,z,\bar{z}) = \sum_n u^n \mathcal{G}^+_{3+n}(z,\bar{z}) \quad \Longrightarrow$

$$P = P(x) = P(t,z,\bar{z})$$

$$\oint du_1 u_1^{s_1} \oint du_2 u_2^{s_1} \lim_{c\to 0} T^+(cu_1,\vec{z}_1)T^+(cu_2,\vec{z}_2) \overset{?}{\sim} \underbrace{\mathcal{G}^+_{2-s_1}(\vec{z}_1)\mathcal{G}^+_{2-s_2}(\vec{z}_2)}$$

2d OPE block of conformal primary gravitons



Caveats

- unlike in 2d CFT, these are <u>not universal</u>, meaning that the 3d OPE blocks of eg. the stress tensor can receive contributions from other primaries

  → we considered contributions to the blocks from specific primaries, eg. $TO \sim O, \;\; TT \sim T$

- Spinning three-point functions in 3d CFT depend only on certain conformally covariant structures, eg.

[Costa, Penedones, Poland, Rychkov '11]

$$\langle T_1 T_2 \mathcal{O}_{\Delta,\ell} \rangle = \frac{\sum_{i=1}^{10} \alpha_i G_\ell^{(i)}(P_i; Z_i)}{(-P_1 \cdot P_2)^{\frac{3}{2}}(-P_1 \cdot P_3)^{\frac{3}{2}}(-P_2 \cdot P_3)^{\frac{3}{2}}}$$

  → conservation of 1, 2 reduces the number of independent coefficients to 2, but all structures still contribute

  → all (position space) spinning blocks can be expressed as weight-shifting operators acting on the scalar blocks

  → Carrollian limit leads to drastic simplifications

Carrollian limits of scalar OPE blocks

$$O_{\Delta_1}(P_1)O_{\Delta_2}(P_2) = \sum_{\Delta_3} \mathcal{N}_S \int d^3P_3 \langle O_{\Delta_1}(P_1)O_{\Delta_2}(P_2)O^S_{3-\Delta_3}(P_3)\rangle O_{\Delta_3}(P_3) + \cdots$$

- sl(2,C) (celestial) primaries can be obtained from so(3,2) ones by Carrollian limit $t = cu, \quad c \to 0$ + dimensional reduction via $\int du u^s$
- dimensional reduction of 3d block $\Longleftrightarrow$ dimensional reduction of 3-point function

  upon expressing O_{Δ_3} in terms of Laurent modes in t_3: $O_{\Delta_3}(t_3, \vec{z}_3) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty} d\lambda (-1)^\lambda \frac{\pi}{\sin\pi(\lambda-1)}(t_3+i\epsilon)^{\lambda-1}O^{(\lambda-1)}_{\Delta_3}(\vec{z}_3)$

- 3-point function of sl(2,C) modes: $\langle \mathcal{O}_{\Delta_1-s_1-1}(\vec{z}_1)\mathcal{O}_{\Delta_2-s_2-1}(\vec{z}_2)\mathcal{O}_{\Delta_3-s_3-1}(\vec{z}_3)\rangle = \prod_{i=1}^{3}\left(\int du_i u_i^{s_i} \lim_{c\to 0} N(\Delta_i, s_i, c)\right)\langle O_{\Delta_1}(cu_1, \vec{z}_1)O_{\Delta_2}(cu_2, \vec{z}_2)O_{\Delta_3}(cu_3, \vec{z}_3)\rangle$

$$\langle O_{\Delta_1}(t_1, \vec{z}_1)O_{\Delta_2}(t_2, \vec{z}_2)O_{\Delta_3}(t_3, \vec{z}_3)\rangle = \frac{c_{123}}{\left(-t_{12}^2 + |z_{12}|^2 + i\epsilon\right)^{\beta_{12}}\left(-t_{13}^2 + |z_{13}|^2 + i\epsilon\right)^{\beta_{13}}\left(-t_{23}^2 + |z_{23}|^2 + i\epsilon\right)^{\beta_{23}}}$$

Carrollian limits of scalar OPE blocks

$$\langle \mathcal{O}_{\Delta_1-s_1-1}(\vec{z}_1)\mathcal{O}_{\Delta_2-s_2-1}(\vec{z}_2)\mathcal{O}_{\Delta_3-s_3-1}(\vec{z}_3)\rangle = \prod_{i=1}^{3}\left(\int du_i u_i^{s_i}\lim_{c\to 0} N(\Delta_i, s_i, c)\right)\langle O_{\Delta_1}(cu_1,\vec{z}_1)O_{\Delta_2}(cu_2,\vec{z}_2)O_{\Delta_3}(cu_3,\vec{z}_3)\rangle$$

- Direct Laurent/Taylor expansion on RHS will be dominated by singular contribution (electric Carroll sector)

- Access magnetic Carroll sector by using the integral representation:

$$\langle O_{\Delta_1}(x_1)O_{\Delta_2}(x_2)O_{\Delta_3}(x_3)\rangle = c_{123}\frac{i^{-\frac{\Delta_1+\Delta_2+\Delta_3+1}{2}}}{\Gamma(\beta_{12})\Gamma(\beta_{13})\Gamma(\beta_{23})}\prod_{i=1}^{3}\left(\int_0^\infty d\alpha_i\alpha_i^{\Delta_i-1}\right)\alpha^{-\frac{\Delta_1+\Delta_2+\Delta_3}{2}}\left(\frac{\alpha}{\pi}\right)^{3/2}\int dt\int d^2z e^{i\sum_{i=1}^{3}\alpha_i(-(t-t_i)^2+|z-z_i|^2+i\epsilon)}, \qquad \alpha\equiv\sum_{i=1}^{3}\alpha_i$$

- Setting $t_i \to cu_i$ and taking $c\to 0$, terms linear in t_i will survive in the exponents since $-(t-t_i)^2 = -t(t-2cu_i)+\mathcal{O}(c^2u_i^2)$

Carrollian limits of scalar OPE blocks

Account for normalization and apply the u_i integrals: $\langle \mathcal{O}_{\delta_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2)\mathcal{O}_{\delta_3}(\vec{z}_3)\rangle = \dfrac{\mathscr{C}_{\Delta_1,\Delta_2,\Delta_3}(\delta_1,\delta_2,\delta_3)\lim_{c\to 0} c^{\sum_i \delta_i - 4}}{(|z_{12}|^2 - i\epsilon)^{\frac{\delta_1+\delta_2-\delta_3}{2}}(|z_{13}|^2 - i\epsilon)^{\frac{\delta_1+\delta_3-\delta_2}{2}}(|z_{23}|^2 - i\epsilon)^{\frac{\delta_3+\delta_2-\delta_1}{2}}}$

- Three-point function of sl(2,C) primaries of dimensions $\delta_i \equiv \Delta_i - s_i - 1$
- If c_{123} is the three-point coefficient dual to contact diagram in AdS$_4$: $c_{123} = 2^{\frac{\Delta_1+\Delta_2+\Delta_3}{2}-3} g c^{-1} \dfrac{1}{2\pi^3} \dfrac{\Gamma(\beta_{12})\Gamma(\beta_{13})\Gamma(\beta_{23})\Gamma(\frac{\sum_i \Delta_i - 3}{2})}{\Gamma(\Delta_1 - \frac{1}{2})\Gamma(\Delta_2 - \frac{1}{2})\Gamma(\Delta_3 - \frac{1}{2})}$

then $\mathscr{C}_{\Delta_1,\Delta_2,\Delta_3}(\delta_1,\delta_2,\delta_3) = g\pi^2 2^{\sum_i(\delta_i - \frac{\Delta_i}{2})-1} i^{\sum_i(\Delta_i+s_i)-1} \dfrac{\Gamma(\frac{\delta_1+\delta_2-\delta_3}{2})\Gamma(\frac{\delta_1+\delta_3-\delta_2}{2})\Gamma(\frac{\delta_2+\delta_3-\delta_1}{2})\Gamma\left(\frac{\sum_i \Delta_i - 3}{2}\right)}{\Gamma\left(\frac{s_1+s_2+s_3}{2}+2\right)}$

- $\lim\limits_{c\to 0} c^{\sum_i \delta_i - 4} = -2\pi\left(\sum_i \delta_i - 4\right)\delta\left(\sum_i \delta_i - 4\right)$ characteristic to celestial amplitude dual to contact three-point vertex in flat space

Carrollian limits of scalar OPE blocks

- The 3d OPE block leads to a collection of sl(2,C) blocks:

$$\mathcal{O}_{\delta_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2) \supset \int d^2\vec{z}_3 \frac{\mathcal{B}_{\Delta_1\Delta_2\Delta_3}(\delta_1,\delta_2)}{|z_{12}|^{2\delta_1+2\delta_2-4}|z_{13}|^{4-2\delta_2}|z_{23}|^{4-2\delta_1}} \mathrm{Res}_{\delta_3=\delta_1+\delta_2-2}\mathcal{O}_{\delta_3}(\vec{z}_3)$$

$$\mathcal{B}_{\Delta_1\Delta_2\Delta_3}(\delta_1,\delta_2) = (-1)^{\delta_1+\delta_2}\mathcal{S}(\Delta_3)\mathcal{C}_{\Delta_1,\Delta_2,3-\Delta_3}(\delta_1,\delta_2,4-\delta_1-\delta_2)$$

from shadow and CFT → CarrCFT → CCFT

- Next we show that the dimensional reduction of 3d blocks involving a spin-1 current and the stress tensor is related to the scalar case via weight-shifting operators

From scalar to spinning blocks

Will consider the case of spin-scalar-scalar 3d block. This is determined by the unique three-point structure:

$$\langle O^{\ell}_{\Delta_1}(P_1) O_{\Delta_2}(P_2) O_{\Delta_3}(P_3)\rangle = c_{\ell 23} \frac{\left[(Z_1\cdot P_2)(P_3\cdot P_1)-(Z_1\cdot P_3)(P_2\cdot P_1)\right]^{\ell}}{(-P_1\cdot P_2)^{\beta_{12}+\frac{\ell}{2}}(-P_2\cdot P_3)^{\beta_{23}+\frac{\ell}{2}}(-P_3\cdot P_1)^{\beta_{31}+\frac{\ell}{2}}} = c_{\ell 23} W_{\ell O}\langle O_{\Delta_1}(P_1) O_{\Delta_2}(P_2) O_{\Delta_3}(P_3)\rangle_{\text{u.n.}}$$

weight-shifting operator

- Set $\ell = 1$, $\Delta_1 = 2$:

$$J^+(x_1) O_\Delta(x_2) = \int d^3x_3 W_{JO}(\Delta_2,\Delta_3)\langle O_{\Delta_1=2}(x_1) O_{\Delta_2}(x_2) O_{\Delta_3}(x_3)\rangle \,|_{\Delta_2=\Delta, \Delta_3 = 3-\Delta}\, O_\Delta(x_3)$$

$$W_{JO}(\Delta_2,\Delta_3) = c_{JO\widetilde{O}} \mathscr{D} \frac{1}{c_{2\Delta_2\Delta_3}} = \sqrt{2} n_J(\Delta)\left(\left(\Delta_2-\frac{1}{2}\right)\left(\beta_{13}-\frac{1}{2}\right)\bar{z}_{12} e^{\partial_{\Delta_2}} - \left(\Delta_3-\frac{1}{2}\right)\left(\beta_{12}-\frac{1}{2}\right)\bar{z}_{13} e^{\partial_{\Delta_3}}\right)$$

does not depend on t, so we can relate to dimensional reduction of scalar block!

From scalar to spinning blocks

- Account for Carrollian/celestial normalization: $W_{JO} \to N_1^{\ell=1} N_2 W_{JO} \frac{1}{N_1 N_2} \propto \mathscr{W}_{JO} \implies$

$$\mathcal{J}^+_{1-s_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2) \propto \int d\delta_3 \int d^2\vec{z}_3 \left[\mathscr{W}_{JO} \frac{\mathscr{C}_{2,\Delta_2,\Delta_3}(\delta_1, \delta_2, \delta_3)}{|z_{12}|^{\delta_1+\delta_2-\delta_3} |z_{13}|^{\delta_1+\delta_2-\delta_3} |z_{23}|^{\delta_2+\delta_3-\delta_1}} \right]_{\Delta_2=\Delta, \Delta_3=3-\Delta} \delta(-s_1+\delta_2+\delta_3-2)(-s_1+\delta_2+\delta_3-2)\mathcal{O}_{2-\delta_3}(\vec{z}_3)$$

- Integral over δ_3 picks up the residue of $\mathcal{O}_{2-\delta_3}$ at $\delta_3 = 2 - \delta_2 + s_1$

- Leading pole in z_{12} for fixed $\bar{z}_{12}$ computed by setting $z_3 = z_2 + t z_{12}, \quad \bar{z}_3 = \bar{z}_2 + \bar{t}\bar{z}_{12}$ [Guevara, Himwich, Pate, Strominger '21]

$$\mathcal{J}^+_{1-s_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2) = \frac{1}{z_{12}} c^{2d}_{JO}(s_1, \delta_2) \int_0^1 d\bar{t}(1-\bar{t})^{\delta_2-1}\bar{t}^{-s_1-1} \mathrm{Res}_{\delta_3=\delta_2-s_1} \mathcal{O}_{\delta_3}(z_2, \bar{z}_2 + \bar{t}\bar{z}_{12}) + \mathcal{O}(z_{12}^0)$$

I. Towers of soft theorems

$$\mathcal{J}^+_{1-s_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2) = \frac{1}{z_{12}} c^{2d}_{JO}(s_1,\delta_2) \int_0^1 d\bar{t}(1-\bar{t})^{\delta_2-1}\bar{t}^{-s_1-1}\mathrm{Res}_{\delta_3=\delta_2-s_1}\mathcal{O}_{\delta_3}(z_2,\bar{z}_2+\bar{t}\bar{z}_{12}) + \mathcal{O}(z^0_{12})$$

$$(-1)^{1-s_1+\delta_2-\Delta}\frac{ign_J(\Delta)\cos\pi\Delta}{2\sqrt{2}(2\Delta-3)\sin\pi s_1\sin\pi\delta_2}$$

- This is the same as the conformal primary version of the tower of soft photon/gluon theorems in 4d Mink space
- The celestial operators identified with $\mathrm{Res}_{\delta\in\mathbb{Z}\_}\mathcal{O}_\delta$ $\Longrightarrow$ guess for CFT-CarrCFT/CCFT normalization not quite right - why??

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- Straightforward generalization to stress tensor-scalar-scalar block along the same lines:

$$\mathcal{T}^{+}_{2-s_1}(\vec{z}_1)\mathcal{O}_{\delta_2}(\vec{z}_2) = \frac{\bar{z}_{12}}{z_{12}} c^{2d}_{TO}(s_1, \delta_2) \int d\bar{t}(1-\bar{t})^{\delta_2}\bar{t}^{-s_1}\mathrm{Res}_{\delta_3=2+\delta_2-s_1}\mathcal{O}_{\delta_3}(z_2, \bar{z}_2 + \bar{t}\bar{z}_{12}) + \mathcal{O}(z_{12}^0)$$

$\Longleftrightarrow$ conformal primary version of the tower of soft graviton theorems in 4d Mink space

II. Infinite symmetry algebras

- The s-algebra of celestial CFT follows from the JJ ~ J OPE block determined by the three-point function

$$\langle J(P_1, Z_1) J(P_2, Z_2) \widetilde{J}(P_3, Z_3) \rangle = c_{JJ\widetilde{J}} \frac{2V_1V_2V_3 + V_1H_{23} + V_2H_{13}}{(-P_1 \cdot P_2)^{3/2}(-P_1 \cdot P_3)^{1/2}(-P_2 \cdot P_3)^{1/2}}$$

where V and H are the building blocks of spinning three-point functions:

$$V_i = \frac{(Z_i \cdot P_j)(P_i \cdot P_k) - (Z_i \cdot P_k)(P_i \cdot P_j)}{\sqrt{(-P_i \cdot P_j)(-P_i \cdot P_k)(-P_j \cdot P_k)}}, \quad H_{ij} = Z_i \cdot Z_j - \frac{(Z_i \cdot P_j)(Z_j \cdot P_i)}{P_i \cdot P_j}, \quad i, j \in \{1,2,3\}$$

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- Consider the three-point function dual to positive helicity gluons in AdS_4: $Z_1 = Z_1^+, \quad Z_2 = Z_2^+, \quad Z_3 = Z_3^-$
- $H_{13}^{+-}\propto t_{13}^2, \quad H_{23}\propto t_{23}^2$ are suppressed in the Carrollian limit
- $V_1^+V_2^+V_3^- \propto V_1^+ = \bar{z}_{12}e^{\partial_{\Delta_2}} - \bar{z}_{13}e^{\partial_{\Delta_3}} \implies \mathcal{J}^+_{1-s_1}(\vec{z}_1)\mathcal{J}^+_{1-s_2}(\vec{z}_2) = \frac{1}{z_{12}}c^{2d}_{JJ}(s_1,s_2)\int d\bar{t}(1-\bar{t})^{-s_2-1}\bar{t}^{-s_1-1}\mathrm{Res}_{\delta_3=1-s_1-s_2}\mathcal{J}^+_{\delta_3}(z_2,\bar{z}_2+\bar{t}\bar{z}_{12}) + \mathcal{O}(z_{12}^0)$

$\iff$ sl(2,C) blocks leading to s-algebra

II. Infinite symmetry algebras

- The $w_{1+\infty}$ algebra of celestial CFT follows from the TT ~ T OPE block determined by the three-point function

$$\langle T(P_1)T(P_2)\widetilde{T}(P_3)\rangle = \lim_{\epsilon\to 0} \frac{\sum_{i=1}^{10} \alpha_i G^{(i)}_{\ell=2}(P_i; Z_i)}{(-P_1\cdot P_2)^3(-P_1\cdot P_3)^\epsilon(-P_2\cdot P_3)^\epsilon}$$

  where G are constructed from the V and H building blocks introduced before

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- Consider $T^+T^+ \sim T^+$ dual to positive helicity gravitons in AdS_4
- To leading order in the Carrollian limit only three structures appear: $(V_1^+)^2(V_2^+)^2(V_3^-)^2$, $H_{12}^{++}V_1^+V_2^+(V_3^-)^2$, and $(H_{12}^{++})^2(V_3^-)^2$
- Translate to weight-shifting operators whose actions on the scalar three-point function are all $\propto (V_1^+)^2(V_2^+)^2(V_3^-)^2$

[cf. double copy relations]

$$\Longrightarrow \mathcal{T}^+_{2-s_1}(\vec{z}_1)\mathcal{T}^+_{2-s_2}(\vec{z}_2) = \frac{\bar{z}_{12}}{z_{12}} c^{2d}_{TT}(s_1,s_2)\int d\bar{t}(1-\bar{t})^{-s_2}\bar{t}^{-s_1}\mathrm{Res}_{\delta_3=4-s_1-s_2}\mathcal{T}^+_{\delta_3}(z_2,\bar{z}_2+\bar{t}\bar{z}_{12}) + \mathcal{O}(z_{12}^0)$$

$\Longleftrightarrow$ sl(2,C) blocks leading to $w_{1+\infty}$-algebra

Summary of results & future directions

Input: (3d) CFT OPE block of scalars, spin-1 currents, stress tensor

↓

Flat space/Carrollian limit + projection onto sl(2,C) modes

↓

Towers of (2d) conformally soft photon/gluon and graviton theorems + associated s- and w-algebras

- Subleading corrections in the Carrollian limit vs. deformations of the algebra?
- Insights into ``celestial OPE'' from associative 3d OPE?
- Predictions for scattering amplitudes from 3d CFT bootstrap (loops, matter)?
- Towards full picture of AFS/CCFT from AdS/CFT including the bulk [to appear sometime this year, w. Núria Navarro]