Flat Space/Carrollian Limit of AdS/CFT in General Dimensions

Romain Ruzziconi

Center for the Fundamental Laws of Nature & Black Hole Initiative
Harvard University

Centre de Physique Théorique
École Polytechnique

Marie Skłodowska-Curie Actions – Global Fellowship "UniFlatHolo No. 101104845"

Based on 2312.10138, 2406.19343, 2504.10291, 2508.06602

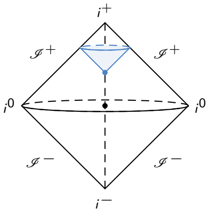
in collaboration with

Luis Fernando Alday, Harshal Kulkarni, Arthur Lipstein,
Lionel Mason, Maria Nocchi, Akshay Yelleshpur Srikant

AdS/CFT meets Carrollian & celestial holography
ICMS, Bayes Centre, Edinburgh, September 9, 2025

How to formulate flat space holography?

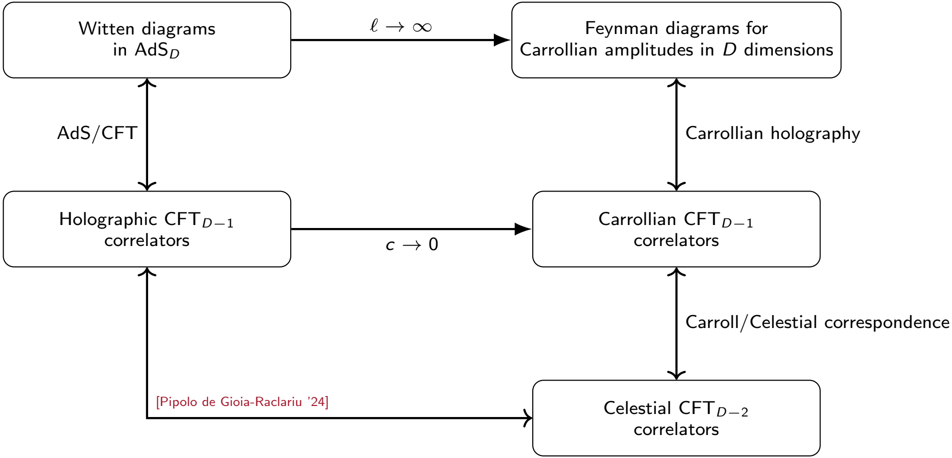
- Correspondence between **gravity in asymptotically flat spacetimes** and a **lower-dimensional field theory** without gravity.
- Two proposals for flat space holography in 4d:
  $\Longrightarrow$ **Celestial holography**: the dual theory is a **2d CFT** living on the **celestial sphere** S^2.
  [de Boer-Solodukhin '03] [He-Mitra-Strominger '15] [Kapec-Mitra-Raclariu-Strominger '16] [Cheung-de la Fuente-Sundrum '16] [Pasterski-Shao-Strominger '17] [Pasterski-Shao '17] [Donnay-Puhm-Strominger '18] [Stieberger-Taylor '18] [Pate-Raclariu-Strominger-Yuan '19] [Adamo-Mason-Sharma '21] ...
  $\Longrightarrow$ **Carrollian holography**: the dual theory is a **3d Carrollian CFT** living at **null infinity** $\mathscr{I} \simeq \mathbb{R} \times S^2$.
  [Arcioni-Dappiaggi '03] [Dappiaggi-Moretti-Pinamonti '06] [Barnich-Compère '07] [Bagchi '10] [Barnich '12] [Bagchi-Detournay-Fareghbal-Simon '12] [Barnich-Gomberoff-Gonzalez '12] [Bagchi-Basu-Grumiller-Riegler '15] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Donnay-Fiorucci-Herfray-Ruzziconi '22] ...
- The two proposals are **equivalent** [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22].



- Main interest of flat space holography: asymptotically flat spacetimes are models for the real world.
  E.g. Collider physics, Astrophysics below the cosmological scale, Gravitational waves, black holes...
  $\Longrightarrow$ For this reason, most of the research has been focused in 4d.
- Evidence suggests that Carrollian holography can be obtained in the flat space ($\ell \to \infty$) / Carrollian limit ($c \to 0$) of AdS/CFT.
  [Bagchi '10] [Barnich-Gomberoff-Gonzalez '12] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Compère-Fiorucci-Ruzziconi '19]
- In $D = 4$: *M-theory on $AdS_4 \times S^7$ dual to $\mathcal{N} = 8$ ABJM theory in 3d*. [Aharony, Bergman, Jafferis, Maldacena '08]
  $\Longrightarrow$ Take the flat / space Carrollian limit [Lipstein-Ruzziconi-Yelleshpur Srikant '25] [Akshay's talk] .
- However, the best-known example of AdS/CFT is in $D = 5$:
  Type IIB String theory on $AdS_5 \times S^5$ dual to $\mathcal{N} = 4$ Super-Yang-Mills in 4d. [Maldacena '97]
  $\Longrightarrow$ Need to extend Carrollian holography and the flat space / Carrollian limit in general dimension D.
  [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

Discuss the flat space / Carrollian limit of AdS/CFT in D dimensions

Summary of the talk



[Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

Bondi coordinates

Can we encode the bulk $\mathcal{S}$-matrix into boundary Carrollian CFT correlators?

- Flat Bondi coordinates $\{u, r, \mathbf{x}\}$ $(u, r \in \mathbb{R},\ \mathbf{x} \in \mathbb{R}^{D-2})$:

$$X^\mu = \frac{u\,\Box_{\mathbf{x}} q^\mu}{D-2} + r\, q^\mu, \quad q^\mu(\mathbf{x}) = \frac{1}{2}\left(1+|\mathbf{x}|^2, 2\mathbf{x}, 1-|\mathbf{x}|^2\right).$$

- Minkowski metric: $ds^2_{\mathbb{R}^{D-1,1}} = -2du\,dr + r^2\,|d\mathbf{x}|^2$.
- Induced Carrollian structure at future/past null infinity $\mathscr{I}^\pm = \{r \to \pm\infty\}$:

$$ds^2_{\mathscr{I}} = q_{ab}dx^a dx^b = 0du^2 + |d\mathbf{x}|^2\,, \qquad n^a\partial_a = \partial_u$$

with $x^a = (u, \mathbf{x})$ the boundary coordinates. [Penrose '63] [Geroch '77] [Ashtekar '14]

- $\text{BMS}_D \simeq$ conformal Carroll symmetries in $D-1$ dimensions:
[Duval-Gibbons-Horvathy '14]

$$\bar{\xi}^a\partial_a = \left[\mathcal{T} + \frac{u}{D-2}\partial_A\mathcal{Y}^A\right]\partial_u + \mathcal{Y}^A\partial_A, \qquad \mathbf{x} = (x^A)$$

with defining property: $\mathcal{L}_{\bar{\xi}} q_{ab} = 2\alpha q_{ab}\,, \mathcal{L}_{\bar{\xi}} n^a = -\alpha n^a\,, \alpha = \frac{1}{D-2}\partial_A\mathcal{Y}^A$.

1. $\mathcal{T} = \mathcal{T}(\mathbf{x})$ is the supertranslation parameter;
2. $\mathcal{Y}^A$ are the Lorentz parameters on S^{D-2}.

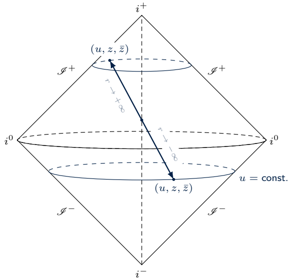


Figure: In $D = 4$, we have $\mathbf{x} = (z, \bar{z})$.

- Scalar conformal Carrollian primary field $\Phi_\Delta(u, \mathbf{x})$ in $D-1$ dimensions:
  [Bagchi-Basu-Kakkar-Mehra '16] [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Nguyen-West '23]

$$\delta_{\bar{\xi}}\Phi_\Delta = \left[\left(\mathcal{T} + \frac{u}{D-2}\partial_A \mathcal{Y}^A \right) \partial_u + \mathcal{Y}^A \partial_A + \frac{\Delta}{D-2}\partial_A \mathcal{Y}^A \right] \Phi_\Delta, \qquad \Delta\text{: conformal dimension.}$$

  (analogue of scalar primary field in CFT_{D-1})

- Comes from the conditions:

$$\underbrace{[K, \Phi_\Delta(0)] = 0, \quad [K_A, \Phi_\Delta(0)] = 0}_{\text{Primary conditions}}, \quad [D, \Phi_\Delta(0)] = -i\Delta\Phi_\Delta(0), \quad \underbrace{[J_{AB}, \Phi_\Delta(0)] = 0}_{\text{Scalar}}, \quad \underbrace{[B_A, \Phi_\Delta(0)] = 0}_{\text{Singlet under boosts}}$$

  where J_{AB}, B_A, K_A, K, D are the generators of the global conformal Carrollian algebra stabilizing the origin.

- Carrollian correlators living at $\mathscr{I}$: $\langle \Phi_{\Delta_1}(u_1, \mathbf{x}_1) \dots \Phi_{\Delta_n}(u_n, \mathbf{x}_n) \rangle$.

Relation with the bulk?

- Strategy: start form a massless scalar field flat space in D dimensions, $\phi(X)$, and push it to $\mathscr{I}$.

$$\Box\phi(X) = 0, \qquad \phi(X) = \int \frac{d^{D-1}\vec{p}}{(2\pi)^{D-1}2p^0}\left[a(\vec{p})e^{ip^\mu X_\mu} + a(\vec{p})^\dagger e^{-ip^\mu X_\mu}\right]$$

- Since $p^\mu p_\mu = 0$, use $p^\mu = \omega q^\mu(\mathbf{x}) = \frac{\omega}{2}\left(1+|\mathbf{x}|^2, 2\mathbf{x}, 1-|\mathbf{x}|^2\right)$.
- Carrollian primary = boundary value of bulk field:
  [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

$$\boxed{\begin{aligned} \Phi_\Delta^{\epsilon=+1}(u,\mathbf{x}) &\sim \lim_{r\to+\infty}\left(r^{\frac{D-2}{2}}\phi(u,r,\mathbf{x})\right) \text{ at } \mathscr{I}^+, \\ \Phi_\Delta^{\epsilon=-1}(u,\mathbf{x}) &\sim \lim_{r\to-\infty}\left(r^{\frac{D-2}{2}}\phi(u,r,\mathbf{x})\right) \text{ at } \mathscr{I}^- . \end{aligned}}$$

$\Longrightarrow$ $\Phi(u,\mathbf{x})$ corresponds to a Carrollian primary of dimension $\Delta = \frac{D-2}{2}$.
$\Longrightarrow$ Here $\epsilon = \pm 1$ if outgoing/incoming.

- Using the stationary phase approximation:

$$\Phi^\epsilon(u,\mathbf{x}) = \int_0^\infty \frac{d\omega}{2\pi}\,(-i\epsilon\omega)^{\frac{D-4}{2}}\,a^\epsilon(\omega,\mathbf{x})e^{-i\epsilon\omega u\omega}$$

with $a^{+1} = a$ and $a^{-1} = a^\dagger$.

- n-point massless scattering amplitude:

$$\mathcal{A}_n\left(p_1^\mu,\dots,p_n^\mu\right)\equiv\mathcal{A}_n\left(\{\omega_1,\mathbf{x}_1\}_{J_1}^{\epsilon_1},\dots,\{\omega_n,\mathbf{x}_n\}_{J_n}^{\epsilon_n}\right)\equiv{}_{\text{out}}\langle 0|a^{\epsilon_1}\left(\omega_1,\mathbf{x}_1\right)\dots a^{\epsilon_n}\left(\omega_n,\mathbf{x}_n\right)|0\rangle_{\text{in}}.$$

after using the parametrization $p^\mu(\omega,\mathbf{x})=\epsilon\omega q^\mu(\omega,\mathbf{x})$.

- Carrollian correlators = scattering amplitudes in position space at $\mathscr{I}$:

$$\boxed{\langle\Phi^{\epsilon_1}(u_1,\mathbf{x}_1)\dots\Phi^{\epsilon_n}(u_n,\mathbf{x}_n)\rangle=\int_0^\infty\prod_{k=1}^n\frac{d\omega_k}{2\pi}\left(-i\epsilon_k\omega_k\right)^{\frac{D-4}{2}}e^{-i\epsilon_k u_k\omega_k}\mathcal{A}_n\left(\{\omega_i,\mathbf{x}_i\}^{\epsilon_i}\right)}$$

$\Longrightarrow$ Amplitudes in position space at $\mathscr{I}$ = Carrollian amplitudes.

[Donnay-Fiorucci-Herfray-Ruzziconi '22] [Mason-Ruzziconi-Yelleshpur Srikant '23] [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

- Extrapolate dictionary for Carrollian holography:

$$\langle\Phi^{\epsilon_1}(u_1,\mathbf{x}_1)\dots\Phi^{\epsilon_n}(u_n,\mathbf{x}_n)\rangle\sim\lim_{r_i\to\epsilon_i\infty}\langle r_1^{\frac{D-2}{2}}\phi^{(s_1)}(u_1,r_1,\mathbf{x}_1)\dots r_n^{\frac{D-2}{2}}\phi^{(s_n)}(u_n,r_n,\mathbf{x}_n)\rangle\,.$$

- Remark: Correlators of Carrollian primaries with $\Delta=\frac{D-2}{2}$ are enough to encode the bulk $\mathcal{S}$-matrix.

Modified Mellin transform

- If $\Phi_\Delta(u,\mathbf{x})$ is a Carrollian primary, then $\partial_u^m \Phi_\Delta(u,\mathbf{x})$ is also a Carrollian primary with shifted conformal dimension $\Delta \to \Delta + m$.
- Start from Carrollian amplitudes and consider correlators of primary-descendants:

$$
\begin{aligned}
\langle \partial_{u_1}^{m_1} \Phi^{\epsilon_1}(u_1,\mathbf{x}_1)\dots \partial_{u_n}^{m_n} \Phi^{\epsilon_n}(u_n,\mathbf{x}_n)\rangle &= \int_0^\infty \prod_{k=1}^n \frac{d\omega_k}{2\pi}\, e^{-i\epsilon_k u_k \omega_k} (-i\epsilon_k \omega_k)^{\frac{D-4}{2}+m_k} \mathcal{A}_n(\{\omega_i,\mathbf{x}_i\}^{\epsilon_i}) \\
&= \int_0^\infty \prod_{k=1}^n \frac{d\omega_k}{2\pi}\, e^{-i\epsilon_k u_k \omega_k} (-i\epsilon_k \omega_k)^{\Delta_k - 1} \mathcal{A}_n(\{\omega_i,\mathbf{x}_i\}^{\epsilon_i}) \\
&\equiv \langle \Phi^{\epsilon_1}_{\Delta_1}(u_1,\mathbf{x}_1)\dots \Phi^{\epsilon_n}_{\Delta_n}(u_n,\mathbf{x}_n)\rangle
\end{aligned}
$$

  where $\Phi_\Delta \equiv \partial_u^m \Phi$ and $\Delta \equiv \frac{D-2}{2} + m$.
  $\implies$ Related to the Modified Mellin transform used to regularize Mellin transform of graviton amplitudes.
  [Banerjee '18] [Banerjee-Ghosh-Pandey-Saha '20]

- Redundant to encode the $\mathcal{S}$-matrix, but can be useful to get rid of IR divergences in Carrollian amplitudes.
- Also very efficient to relate Carrollian with celestial holography in D dimensions: if $u_k = 0\ \forall k$, we recover the Mellin transform defining celestial amplitudes.
  [Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22]
  $\implies$ Used in [Kulkarni-Ruzziconi-Yelleshpur Srikant '25] to match with [Pipolo de Gioia-Raclariu '24].

- Amplitudes in position space at $\mathscr{I}$ = Carrollian amplitudes.
- Consistent with the (global) conformal Carrollian Ward identities.
  $\Longrightarrow$ The low-point correlation functions are completely fixed by the symmetries.
- In particular, for the 2-point function [Chen-Liu-Zheng, '21]:

$$\langle \Phi_{\Delta_1}(u_1,\mathbf{x}_1)\Phi_{\Delta_2}(u_2,\mathbf{x}_2)\rangle = \begin{cases} \frac{\alpha}{(u_1-u_2)^{\Delta_1+\Delta_2-(D-2)}}\delta^{(D-2)}(\mathbf{x}_1-\mathbf{x}_2) & \text{(Electric branch)} \\ \frac{\beta}{|\mathbf{x}_1-\mathbf{x}_1|^{\Delta_1+\Delta_2}}\delta_{\Delta_1,\Delta_2} & \text{(Magnetic branch)} \end{cases}$$

$\Longrightarrow$ Electric branch relevant for massless scattering.

[Donnay-Fiorucci-Herfray-Ruzziconi '22] [Bagchi-Banerjee-Basu-Dutta '22] [Mason-Ruzziconi-Yelleshpur Srikant '23]

Two-point Carrollian amplitude

- Two-point amplitude in D dimensions:

$$\mathcal{A}_2 = 2\kappa_2 p_1^0 \delta^{D-1}(p_1 + p_2) = \frac{2\kappa_2}{\omega_1^{D-3}} \delta^{D-2}(\mathbf{x}_{12})\, \delta(\omega_1 - \omega_2)\, \delta_{\epsilon_1, -\epsilon_2},$$

where $\omega_{12} = \omega_1 - \omega_2$ and $\mathbf{x}_{12} = \mathbf{x}_1 - \mathbf{x}_2$.

- Two-point Carrollian amplitude in D dimensions [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]:

$$\begin{aligned}\langle \Phi^{\epsilon_1}_{\Delta_1, J_1}(u_1, \mathbf{x}_1)\, \Phi^{\epsilon_2}_{(\Delta_2, J_2)}(u_2, \mathbf{x}_2)\rangle &= \int_0^\infty \prod_{k=1}^{2} \frac{d\omega_k}{2\pi} (-i\epsilon_k\omega_k)^{\Delta_k - 1} e^{-i\epsilon_k u_k \omega_k - \varepsilon\omega_k} \mathcal{A}_2 \\ &= \frac{\kappa_2}{2\pi^2} i^D (-1)^{\Delta_1} \frac{\delta^{D-2}(\mathbf{x}_{12})\, \Gamma(\Delta_1 + \Delta_2 - (D-2))}{(u_{12} - i\epsilon_1\varepsilon)^{\Delta_1 + \Delta_2 - (D-2)}} \delta_{\epsilon_1, -\epsilon_2}\end{aligned}$$

$\Longrightarrow$ Agrees with the solution of the Carrollian Ward identities (electric branch). ✓

- Useful example for later: four-point contact diagram $\mathcal{A}_{4,c} = \kappa_4 \delta^{(D)}(p_1 + p_2 + p_3 + p_4)$ for a massless scalar.
- Using an appropriate parametrization for the momentum conserving δ-function, the four integrals can be computed [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]:

$$\langle \Phi^{\epsilon_1}_{\Delta_1}(u_1, z_1, \bar{z}_1) \dots \Phi^{\epsilon_4}_{\Delta_4}(u_4, z_4, \bar{z}_4) \rangle_c = \frac{\kappa_4}{8\,(2\pi)^4} \frac{\mathcal{S}\,\mathcal{Z}\,(\mathbf{x}_{ij})}{\mathcal{U}^{\Sigma_\Delta - D}} \boxed{\delta\,(z - \bar{z}) \prod_{j=5}^{D} \delta\,(q_4 \cdot n_j)}$$

$$\times\, (-1)^{\Delta_1 + \Delta_3 - D}\, z^{\Delta_1 - \Delta_2}\, (1 - z)^{\Delta_2 - \Delta_3}\, \Gamma\,(\Sigma_\Delta - D)\,,$$

where $(q_1, \dots, q_4, n_5, \dots, n_D)$ forms a basis of $\mathbf{R}^{D-1,1}$, and the cross-ratios in the celestial sphere in $D - 2$ dimensions are

$$z\bar{z} = \frac{|\mathbf{x}_{12}|^2\, |\mathbf{x}_{34}|^2}{|\mathbf{x}_{13}|^2\, |\mathbf{x}_{24}|^2}, \qquad (1 - z)(1 - \bar{z}) = \frac{|\mathbf{x}_{14}|^2\, |\mathbf{x}_{23}|^2}{|\mathbf{x}_{13}|^2\, |\mathbf{x}_{24}|^2}.$$

Explicit expressions:

$$\mathcal{S} = \Theta\,(-z\epsilon_1\epsilon_4)\, \Theta\,((1 - z)\, z\epsilon_2\epsilon_4)\, \Theta\,((z - 1)\epsilon_3\epsilon_4)\,,$$

$$\mathcal{Z}\,(\mathbf{x}_{ij}) = \frac{\left|\mathbf{x}^2_{14}\right|^{\Delta_3 - 1} \left|\mathbf{x}^2_{24}\right|^{\Delta_1 - 2} \left|\mathbf{x}^2_{34}\right|^{\Delta_2 - 1}}{\left|\mathbf{x}^2_{12}\right|^{\Delta_1 - 1} \left|\mathbf{x}^2_{13}\right|^{\Delta_3} \left|\mathbf{x}^2_{23}\right|^{\Delta_2 - 1}},$$

$$\mathcal{U} = -u_{14} z \frac{|\mathbf{x}_{24}|^2}{|\mathbf{x}_{12}|^2} + u_{24} \frac{1 - z}{z} \frac{|\mathbf{x}_{34}|^2}{|\mathbf{x}_{23}|^2} - u_{34} \frac{1}{1 - z} \frac{|\mathbf{x}_{14}|^2}{|\mathbf{x}_{13}|^2}.$$

- Holographic correlators are boundary CFT correlators computed via AdS Witten diagrams in the bulk.
  $\Longrightarrow$ Take the flat space limit $\ell \to \infty$ in position space.
  $\Longrightarrow$ Carrollian amplitudes are natural objects obtained in the limit.
- See also recent related works: [Pipolo de Gioia-Raclariu '22] [Bagchi-Dhivakar-Dutta '23] [Marotta-Skenderis-Verma '24] [Kraus, Myers '24]
  [Akshay, Ana, Jan, Per, Prateksh, and Tom talks]
- Bondi coordinates $X = (u, r, \mathbf{x})$ also exist in AdS_D:

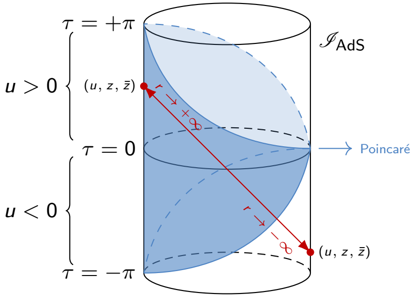
$$ds^2_{\text{AdS}} = -\frac{r^2}{\ell^2} du^2 - 2du\, dr + r^2 \,|d\mathbf{x}|^2 \,.$$

- Relation to Poincaré coordinates $X = (\rho, x^0, \dots, x^{D-2})$:

$$ds^2_{AdS} = \frac{\ell^2}{\rho^2}\left(d\rho^2 - (dx^0)^2 + \sum_{i=1}^{D-2}(dx^i)^2\right) \,.$$

$$\rho = \frac{\ell}{r}\,, \quad x^0 = -\frac{\ell}{r} + \frac{u}{\ell}\,, \quad x^i = \mathbf{x}^i \,.$$

- Advantages of Bondi coordinates:
  Admit a well-defined flat limit + extend beyond Poincaré patch.



Flat-space/Carrollian limit correspondence

- What is the boundary CFT interpretation of taking the flat limit in the bulk $\ell \to \infty$?

  1. Flat limit in the bulk: $ds^2_{AdS} = -\frac{r^2}{\ell^2} du^2 - 2dudr + r^2 |d\mathbf{x}|^2 \implies ds^2_{Flat} = -2dudr + r^2 |d\mathbf{x}|^2$
  2. Carrollian limit at the boundary: $ds^2_{\mathscr{I}_{AdS}} = -\frac{1}{\ell^2} du^2 + |d\mathbf{x}|^2 \implies ds^2_{\mathscr{I}} = 0du^2 + |d\mathbf{x}|^2$

$$\boxed{\text{Flat limit in the bulk } (\ell \to \infty)} \xleftrightarrow{\frac{1}{\ell} \equiv c} \boxed{\text{Carrollian limit at the boundary } (c \to 0)}$$

- This observation extends to asymptotically AdS solutions of Einstein equations.
  [Barnich-Gomberoff-Gonzalez '12] [Ciambelli-Marteau-Petkou-Petropoulos-Siampos '18] [Poole-Skenderis-Taylor '18] [Compère-Fiorucci-Ruzziconi '19] [Campoleoni-Delfante-Pekar-Petropoulos-Rivera Betancour-Vilatte '23]

- Does this correspondence work at the level of the correlators?
  [Alday-Nocchi-Ruzziconi-Yelleshpur Srikant '24] [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

  1. Take the flat space limit of AdS Witten diagrams using bulk Bondi coordinates.
  2. Take the Carrollian limit of holographic CFT correlators from an intrinsic boundary perspective.

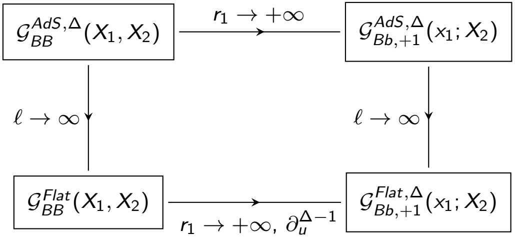
Flat limit of bulk-to-bulk propagator in Bondi coordinates

- Bulk-to-bulk propagator $\mathcal{G}^{AdS,\Delta}_{BB}(X_1, X_2)$ for a massive scalar field in AdS$_D$ solves

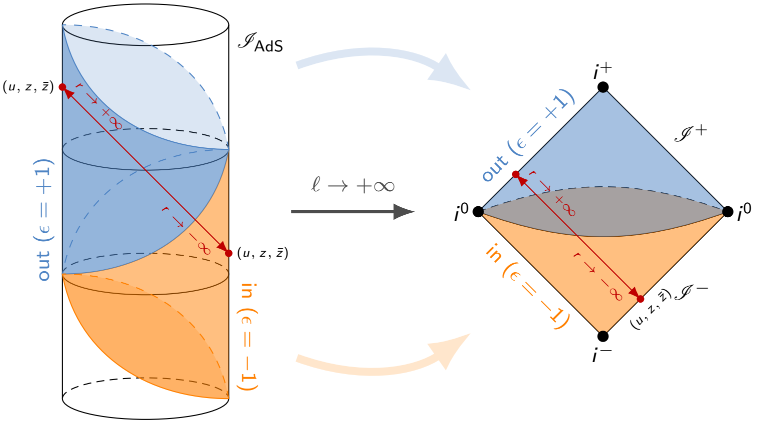
$$(\Box_{X_1} + M^2)\mathcal{G}^{AdS,\Delta}_{BB}(X_1, X_2) = \frac{1}{\sqrt{-g}}\delta^{(4)}(X_{12}),$$

$$\Box = \Big[\underbrace{\frac{D\, r_1}{\ell^2}\partial_{r_1}}_{\to 0 \text{ if } \ell\to\infty} + \frac{r^2}{\ell^2}\partial^2_{r_1} + \frac{\Box_{x_1}}{r_1^2} - \frac{D-2}{r}\partial_u - 2\partial_u\partial_r \Big], \qquad M^2 = \underbrace{\frac{\Delta(D-1-\Delta)}{\ell^2}}_{\to 0 \text{ if } \ell\to\infty}.$$

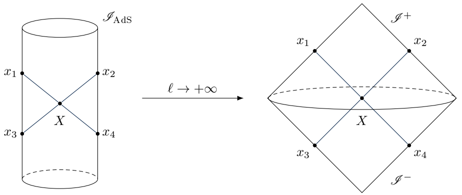
- From massive scalar in AdS to massless scalar in flat space.



- Flat space bulk-to-boundary propagator: $\mathcal{G}^{Flat,\Delta}_{Bb,+} = \int_0^{+\infty} \frac{d\omega}{2\pi}(-i\omega)^{\Delta-1} e^{-i\omega_1 u_1} e^{-ip_1^\mu X_{2\mu}}$. [Prateksh's talk]
- Outgoing ($\epsilon = +1$) / incoming ($\epsilon = -1$) bulk-to-boundary propagator: $r_1 \to +\infty$ / $r_2 \to -\infty$.



[Alday-Nocchi-Ruzziconi-Yelleshpur Srikant '24]



$$\langle \mathcal{O}^{\epsilon_1}_{\Delta_1}(x_1)\mathcal{O}^{\epsilon_2}_{\Delta_2}(x_2)\mathcal{O}^{\epsilon_3}_{\Delta_3}(x_3)\mathcal{O}^{\epsilon_4}_{\Delta_4}(x_4)\rangle_c = \kappa_4 \int_{AdS} d^D X \prod_{i=1}^4 \mathcal{G}^{AdS,\Delta_i}_{Bb,\epsilon_i}(x_i, X) \qquad \text{(four-point contact diagram for scalars)}$$

$$\xrightarrow{\ell\to\infty} \kappa_4 \int_{Flat} d^D X \prod_{i=1}^4 \mathcal{G}^{Flat,\Delta_i}_{Bb,\epsilon_i}(x_i, X) \times \ell^{\sum_i \Delta_i - 2D + 4}$$

$$= \kappa_4 \int_0^{+\infty} \prod_{i=1}^4 \frac{d\omega_i}{2\pi} (-i\epsilon_i\omega_i)^{\Delta_i - 1} e^{-i\epsilon_i\omega_i u_i} \int_{Flat} d^D X e^{-i\sum_i p_i^\mu X_\mu} \times \ell^{\sum_i \Delta_i - 2D + 4}$$

$$= \kappa_4 \int_0^{+\infty} \prod_{i=1}^4 \frac{d\omega_i}{2\pi} (-i\epsilon_i\omega_i)^{\Delta_i - 1} e^{-i\epsilon_i\omega_i u_i} \delta^{(D)}(p_1 + p_2 + p_3 + p_4) \times \ell^{\sum_i \Delta_i - 2D + 4}$$

$$= \langle \Phi^{\epsilon_1}_{\Delta_1}(x_1)\Phi^{\epsilon_2}_{\Delta_2}(x_2)\Phi^{\epsilon_3}_{\Delta_3}(x_3)\Phi^{\epsilon_4}_{\Delta_4}(x_4)\rangle_c \times \ell^{\sum_i \Delta_i - 2D + 4} \quad \Longrightarrow \quad \mathcal{O}^{\epsilon}_{\Delta}(x) \sim \ell^{\Delta - \frac{D-2}{2}} \Phi^{\epsilon}_{\Delta}(x)$$

- Summary:

Boundary: Bulk:

$$\langle \mathcal{O}^{\epsilon_1}_{\Delta_1}(x_1)\mathcal{O}^{\epsilon_2}_{\Delta_2}(x_2)\mathcal{O}^{\epsilon_3}_{\Delta_3}(x_3)\mathcal{O}^{\epsilon_4}_{\Delta_3}(x_4)\rangle_c = \int_{AdS} d^D X \mathcal{G}^{AdS,\Delta_1}_{Bb,\epsilon_1}(x_1, X)\mathcal{G}^{AdS,\Delta_2}_{Bb,\epsilon_2}(x_2, X)\mathcal{G}^{AdS,\Delta_3}_{Bb,\epsilon_3}(x_3, X)\mathcal{G}^{AdS,\Delta_4}_{Bb,\epsilon_4}(x_4, X)$$

$$\Big\downarrow c \to 0 \quad ? \qquad\qquad \Big\downarrow \ell \to \infty \quad \checkmark$$

$$\langle \Phi^{\epsilon_1}_{\Delta_1}(x_1)\,\Phi^{\epsilon_2}_{\Delta_2}(x_2)\,\Phi^{\epsilon_3}_{\Delta_3}(x_3)\,\Phi^{\epsilon_4}_{\Delta_4}(x_4)\rangle_c = \int_{Flat} d^D X \mathcal{G}^{Flat,\Delta_1}_{Bb,\epsilon_1}(x_1, X)\mathcal{G}^{Flat,\Delta_2}_{Bb,\epsilon_2}(x_2, X)\mathcal{G}^{Flat,\Delta_3}_{Bb,\epsilon_3}(x_3, X)\mathcal{G}^{Flat,\Delta_4}_{Bb,\epsilon_4}(x_4, X)$$

$\Longrightarrow$ AdS Witten diagrams reduce to **Feynman diagrams** for Carrollian amplitudes in the $\ell \to \infty$ limit!

Can we reproduce this result by an intrinsic Carrollian limit in the boundary CFT?

Carrollian limit of holographic CFT correlators

- Lorentzian CFT two-point function can be obtained from Euclidean signature by analytic continuation:
- Behaviour of the CFT 2-point function in Lorentzian signature in the Carrollian limit:

$$\langle \mathcal{O}^-_\Delta(x_1)\,\mathcal{O}^+_\Delta(x_2)\rangle = \frac{1}{\left(-c^2 u_{12}^2 + |\mathbf{x}_{12}|^2 + i\varepsilon\right)^{\Delta_1}} \xrightarrow{c\to 0} c^0 \underbrace{\frac{1}{|\mathbf{x}_{12}|^{2\Delta}}}_{\text{Magnetic}} + c^{D-2-2\Delta} \underbrace{\frac{\Gamma\left(\Delta - \frac{D-2}{2}\right)}{\Gamma(\Delta)} \frac{\delta^{(D-2)}(\mathbf{x}_{12})}{\left(-u_{12}^2 + i\varepsilon\right)^{\Delta - \frac{D-2}{2}}}}_{\text{Electric}} .$$

- For $\Delta \geq \frac{D-2}{2} \implies$ Electric branch is leading in the limit $c \to 0$ and is found on the support of the δ-function ($\mathbf{x}_{12} = 0$).
- From CFT primary to Carrollian CFT primary: $\mathcal{O}_\Delta(x) \sim c^{\frac{D-2}{2}-\Delta}\Phi_\Delta(x)$

[Alday-Nocchi-Ruzziconi-Yelleshpur Srikant '24] [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

$$\lim_{c\to 0} c^{2\Delta - D+2}\langle \mathcal{O}_\Delta(x_1)\,\mathcal{O}_\Delta(x_2)\rangle = \langle \Phi_\Delta(x_1)\,\Phi_\Delta(x_2)\rangle \qquad \checkmark$$

Lorentzian four-point function

- Holographic four-point contact diagram in Euclidean signature (see e.g. [Bissi-Sinha-Zhou 22']):

$$\langle \mathcal{O}_{\Delta_1}(x_1)\,\mathcal{O}_{\Delta_2}(x_2)\,\mathcal{O}_{\Delta_3}(x_3)\,\mathcal{O}_{\Delta_4}(x_4)\rangle_E^c = \kappa_4 \int_{AdS_4} d^D X \prod_{i=1}^{4} G_{Bb}^{AdS,\Delta_i}(x_i, X) \propto \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4}(Z,\bar{Z}) \ ,$$

where $Z\bar{Z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$ and $(1-Z)(1-\bar{Z}) = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2}$ are the CFT cross-ratios in $D-1$ dimensions.

- Example:

$$\bar{D}_{1,1,1,1}(Z,\bar{Z}) = \frac{1}{Z-\bar{Z}}\left[2\,\mathrm{Li}_2(Z) - 2\,\mathrm{Li}_2(\bar{Z}) + \log Z\bar{Z} \log\left(\frac{1-Z}{1-\bar{Z}}\right)\right] \ .$$

- Analytic continuation from Euclidean to Lorentzian signature: different possibilities [Gary-Giddings-Penedones '09]
- Some analytic continuations lead to singularities in $Z-\bar{Z}$, e.g. $\bar{D}_{1,1,1,1} \to \bar{D}_{1,1,1,1} + \frac{4\pi^2}{Z-\bar{Z}} + \frac{2\pi i}{Z-\bar{Z}} \log \frac{1-\bar{Z}}{1-Z}$

$\Longrightarrow$ **Bulk-point singularity** [Maldacena-Simmons-Duffin-Zhiboedov '17]

Carrollian limit of the four-point function

- Relations between cross-ratios in $D-1$ and $D-2$ dimensions: $\left(Z-\bar{Z}\right)^2 = (z-\bar{z})^2 + \mathcal{O}(c^2)$.
- Similarly to the 2-point function, the Carrollian limit on the support $(z-\bar{z})^2 = 0$ produces leading contributions in c coming from the bulk-point singularity. [Alday-Nocchi-Ruzziconi-Yelleshpur Srikant '24]
- This looks like one constraint, but it is necessary to impose $D-3$ constraints on the points $\mathbf{x}_1, \ldots, \mathbf{x}_n$.
- Consider the Gram matrix: $G_{ij} = |\mathbf{x}_{ij}|^2 = -2q_i \cdot q_j$ $(i,j = 1,\ldots,4)$. [Kulkarni-Ruzziconi-Yelleshpur Srikant '25]

$$\det(G_{ij}) = \frac{(z-\bar{z})^2}{|\mathbf{x}_{13}|^4 \, |\mathbf{x}_{24}|^4} = 0$$

- This is a positive semidefinite matrix, and $\det(G_{ij}) = 0$ iff the q_is are linearly dependent, i.e. iff there exists c_is such that $\sum_i c_i q_i = 0$. $\Longrightarrow$ You can recover the bulk momentum conserving δ-function in that way.

$$\lim_{c\to 0} c^{\alpha} \bar{D}_{\Delta_1,\Delta_2,\Delta_3,\Delta_4} = \boxed{\delta\left(z-\bar{z}\right) \prod_{j=5}^{D} \delta\left(q_4 \cdot n_j\right)} \times \mathcal{R}(u_{ij}, \mathbf{x}_{ij}) \tag{1}$$

- $\mathcal{R}$ and α are fixed by demanding normalization of the δ-function and having a finite and non-zero limit.
- The choice of analytic continuation fixes the Θ functions (non-trivial Carrollian electric branch on the support of Θ)

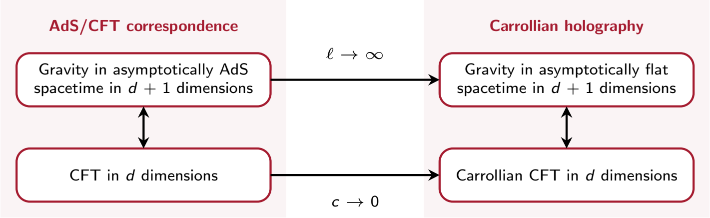
$$\lim_{c\to 0} c^{\Sigma_\Delta - 2D + 4} \langle \mathcal{O}_{\Delta_1}(x_1)\, \mathcal{O}_{\Delta_2}(x_2)\, \mathcal{O}_{\Delta_3}(x_3)\, \mathcal{O}_{\Delta_4}(x_4) \rangle_c = \langle \Phi_{\Delta_1}(x_1)\, \Phi_{\Delta_2}(x_2)\, \Phi_{\Delta_3}(x_3)\, \Phi_{\Delta_4}(x_4) \rangle_c \qquad \checkmark$$

- The above analysis is valid for any scalar subsector of AdS/CFT.
- In $D = 4$: *M-theory on* $AdS_4 \times S^7$ *dual to* $\mathcal{N} = 8$ *ABJM theory in* $3d$. [Akshay's talk]
  [Lipstein-Ruzziconi-Yelleshpur Srikant '25]
- in $D = 5$: *Type IIB String theory on* $AdS_5 \times S^5$ *dual to* $\mathcal{N} = 4$ *Super-Yang-Mills in* $4d$.
  (WIP with Burkhard Eden, Paul Heslop, Harshal Kulkarni, Arthur Lipstein, Akshay Yelleshpur Srikant)
- Subtle point: in the flat space limit, the S^5 decompactifies!

$$ds^2_{AdS_5 \times S^5} = ds^2_{AdS_5} + \ell^2 ds^2_{S^5} \xrightarrow{\ell \to \infty} ds^2_{\mathbb{R}^{9,1}}$$

  $\Longrightarrow$ 10d flat space! but the Carrollian limit leads to a $4d$ Carrollian CFT.
- First option: the correlators of the $4d$ Carrollian CFT are re-interpreted as $10d$ bulk amplitudes restricted to a $5d$ hyperplane (momentum in $\mathbb{R}^{1,4}$ directions, helicity in the transverse $\mathbb{R}^5$ space).
- Second option: emergent symmetries in the Carrollian limit to obtain a $9d$ Carrollian CFT.
  $\Longrightarrow$ Carrollian correlators could reproduce the full $10d$ flat space amplitudes!

- Carrollian holography: $\mathcal{S}$-matrix can be encoded in terms of Carrollian CFT correlators ✓
- Carrollian holography is naturally related to AdS/CFT via the flat limit / Carrollian limit ✓



- Towards a top-down Carrollian hologram...

Thank you!