Proper cocycles, measure equivalence and L_p-Fourier multipliers

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Various properties of group von Neumann algebras are invariant under measure equivalence, such as the Haagerup property and weak amenability. This fact, as observed by Jolissiant, is based on a certain transference of Fourier-Schur multipliers. We establish a new transference method of completely bounded L_p-Fourier multipliers for proper cocycles for pmp group actions on standard probability spaces. This generalizes the aforementioned results by Jolissaint (and Haagerup et al) which only deals with the case $p=\infty$. In particular, this gives a natural transference method of Fourier multipliers between groups with measure equivalence, which directly implies and notably generalizes the main result of Hong-Wang-Wang on the pointwise convergence of noncommutative Fourier series on amenable groups. As a second application, this theory also yields a noncommutative Jodiet theorem that transfers L_p-Fourier multipliers from lattices to the ambient group, which strengthens the previous results for Schur multipliers obtained by Haagerup and Lafforgue-de la Salle. Our approach also provides a proper analogue of noncommutative Hilbert transform on $SL(2,R)$ for all $1<p<\infty$. I will also mention the links with de Leeuw restriction problems. This is ongoing joint work with Gan Yao and Runlian Xia.