High-energy string theory and tensionless strings

MAX-PLANCK-INSTITUT FÜR PHYSIK

Stephan Stieberger, MPP München

AdS/CFT meets Carrollian & Celestial holography
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MODERN FOUNDATIONS of SCATTERING AMPLITUDES

based on:

Xavier Kervyn, St.St.:

High-Energy String Theory and the Celestial Sphere
arXiv:2504.13738, JHEP 09 (2025) 044

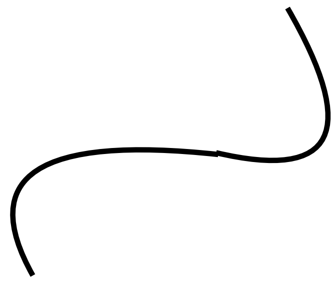
- High energy limit of string theory: subleading corrections to amplitudes

- Amplitudes of tensionless (null) strings

- High energy string limit and celestial sphere

three (un)related topics ?

I. High energy string theory



$T = \frac{1}{2\pi\alpha'}$

T = string tension $\quad \alpha' = M_{\text{string}}^{-2}$

$T \longrightarrow \infty$	$T \longrightarrow 0$
field-theory limit: *very well understood*	high-energy limit: quantum gravity effects strongest *less understood*
discrete spectrum of massive higher spin states (infinite tower of states of mass $\sqrt{n/\alpha'}$, $n \in \mathbf{Z}_+$)	infinite many massless higher spin states
	un-Higgsed phase of string theory infinite many global symmetries mixing all oscillators
expansion at $\alpha' = 0$: effective action, integrate out massive higher spin states	expansion at $\alpha' = \infty$: effective action ? higher-spin theory ?

$T \longrightarrow \infty$	$T \longrightarrow 0$
amplitudes: *nice mathematical structure* Riemann zeta values $\zeta(n)$	amplitudes: *nice mathematical structure* Riemann zeta values $\zeta(-n)$
CFT on the world-sheet	CFT on the world-sheet
localization: pinching string world-sheet	saddle-point approximation: quantum fluctuations around classical solution
open vs. closed strings: KLT, double copy	open vs. closed strings blurred tensionless strings ?

?

High energy representation of string amplitudes

four-point open and closed string (tree-level) amplitudes
in flat background

$$s = \alpha'(p_1 + p_2)^2 \quad , \quad t = \alpha'(p_1 - p_3)^2 \quad , \quad u = \alpha'(p_2 - p_3)^2$$

$$\boxed{\mathscr{A}(1,2,3,4) = \frac{\Gamma(1-s)\Gamma(1-u)}{\Gamma(1+t)}\, A_{YM}(1,2,3,4)}$$

open superstring
tree-level (disk)

$$\underbrace{\frac{\Gamma(1-s)\Gamma(1-u)}{\Gamma(1+t)}} = (-s)\int_0^1 dx\, x^{-s-1}\,(1-x)^{-u}$$

$$\boxed{\mathscr{M} = \pi\,\frac{su}{t}\,\frac{\Gamma(-s)\Gamma(-u)\Gamma(-t)}{\Gamma(s)\Gamma(u)\Gamma(t)}\, A_{YM}(1,2,3,4)\,\tilde{A}_{YM}(1,2,3,4)}$$

closed superstring
tree-level (sphere)

$$\underbrace{\pi\,\frac{su}{t}\,\frac{\Gamma(-s)\Gamma(-u)\Gamma(-t)}{\Gamma(s)\Gamma(u)\Gamma(t)}} = (-s^2)\int_0^1 d^2z\,|z|^{-2s-2}\,|1-z|^{-2u}$$

$\Re s, \Re u < 0,\ \Re t > 0$ with: $s + u + t = 0$

$\alpha' \longrightarrow 0$	$\alpha' \longrightarrow \infty$
$s, u, t \to 0$	$s, u \to -\infty,\ t \to +\infty$

$$\mathscr{A}_0(1,2,3,4) = \exp\left\{\sum_{n=1}^{\infty} \frac{\zeta(2n)}{(2n)} \left(s^{2n} + u^{2n} - t^{2n}\right)\right\} \times \exp\left\{\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{(2k+1)} \left(s^{2k+1} + u^{2k+1} + t^{2k+1}\right)\right\} \times A_{YM}(1,2,3,4)$$

$$\mathscr{M}_0 = \pi \frac{su}{t} \exp\left\{2\sum_{k=1}^{\infty} \frac{\zeta(2k+1)}{(2k+1)} \left(s^{2k+1} + u^{2k+1} + t^{2k+1}\right)\right\} \times A_{YM}(1,2,3,4)\ \tilde{A}_{YM}(1,2,3,4)$$

$$\text{sv}\ \zeta(2k) = 0,$$
$$\text{sv}\ \zeta(2k+1) = 2\ \zeta(2k+1),\ k \geq 1$$

Schlotterer, St.St. (2012)

$$\mathscr{A}_{-\infty}(1,2,3,4) = (2\pi)^{1/2} \left(\frac{su}{t}\right)^{1/2} e^{-s\ln s - u\ln u - t\ln t}\ (-1)^{-s-u} \times \exp\left\{\sum_{k=1}^{\infty} \frac{\zeta(1-2k)}{(2k-1)} \left(\frac{1}{s^{2k-1}} + \frac{1}{u^{2k-1}} + \frac{1}{t^{2k-1}}\right)\right\} \times A_{YM}(1,2,3,4)$$

$$\mathscr{M}^c_\infty = i\pi \frac{su}{t}\ e^{-2s\ln s - 2u\ln u - 2t\ln t} \times \exp\left\{2\sum_{k=1}^{\infty} \frac{\zeta(1-2k)}{(2k-1)} \left(\frac{1}{s^{2k-1}} + \frac{1}{u^{2k-1}} + \frac{1}{t^{2k-1}}\right)\right\} \times A_{YM}(1,2,3,4)\ \tilde{A}_{YM}(1,2,3,4)$$

$$\mathscr{M}^c_\infty = \frac{i}{2}\ \mathscr{A}_{-\infty}(1,2,3,4)\ \cdot\ \tilde{\mathscr{A}}_{-\infty}(1,2,3,4)$$

$$\zeta(1-2k) = -\frac{B_{2k}}{2k} = \frac{(-1)^k}{2^{2k-1}}\ \frac{(2k-1)!}{\pi^{2k}}\ \zeta(2k)\ ,\ k \geq 1$$

Kervyn, St.St. (2025)

$$\sim\ e^{\frac{1}{g+1}\left(-s\ln s - u\ln u - t\ln t\right)}$$

$$s \to +\infty,\ u, t \to -\infty$$

$$\mathscr{A}_{+\infty}(1,2,3,4) = (2\pi)^{1/2} \frac{\sin(\pi t)}{\sin(\pi s)} \left(\frac{su}{t}\right)^{1/2} e^{-s\ln s - u\ln u - t\ln t} (-1)^{-u-t}$$
$$\times \exp\left\{ \sum_{k=1}^{\infty} \frac{\zeta(1-2k)}{(2k-1)} \left(\frac{1}{s^{2k-1}} + \frac{1}{u^{2k-1}} + \frac{1}{t^{2k-1}} \right) \right\} A_{YM}(1,2,3,4)$$

$$\mathscr{M}_{+\infty} = -2\pi \frac{\sin(\pi u)\sin(\pi t)}{\sin(\pi s)} \frac{su}{t} e^{-2s\ln s - 2u\ln u - 2t\ln t}$$
$$\times \exp\left\{ 2\sum_{k=1}^{\infty} \frac{\zeta(1-2k)}{(2k-1)} \left(\frac{1}{s^{2k-1}} + \frac{1}{u^{2k-1}} + \frac{1}{t^{2k-1}} \right) \right\}$$
$$\times A_{YM}(1,2,3,4)\ \tilde{A}_{YM}(1,2,3,4)$$

Remarks

→ we have amplitudes at hand describing corrections (in $1/\alpha'$)
to high-energy limit of string theory
in similar way than low-energy string expansion (in α')

taking into account light higher spin modes
contributing similarly to $\mathscr{A}_\infty, \mathscr{M}_\infty$

ultra high-energy limit $\alpha' \to \infty$ interesting on its own
(opposite to field-theory limit $\alpha \to 0$):

II. Amplitudes of tensionless strings

- system of (infinite many) massless higher spin modes

effective field-theory breaks down (in flat space-time),
but amplitudes with $\zeta(-n)$ contributions at hand
(i.e. non-trivial quite universal functions):

Riemann zeta values of odd negative weights $\zeta(-n)$:

- *appear from string world-sheet*
  as quantum fluctuation
  of saddle point approximation

- *rational numbers*

- *feature zeta function regularization*
  of massless fields of arbitrary spin

- *resurgence properties of infinite sum representations:*
  Borel transform
  (cf. weak/strong-coupling expansion in AdS/CFT)

- $\mathrm{Vir} \otimes \overline{\mathrm{Vir}}$ (Vir) symmetry on closed (open) string world-sheet

- (tree-level) open and closed string amplitudes qualitatively same (in contrast to field-theory limit $\alpha' \to 0$) no distinction between open and closed strings

distinction between open and closed string only for tensile string and becomes blurred in the tensionless limit of string theory

Francia, Mourad, Sagnotti (2007)

Bagchi, Banerjee, Parekh (2019)

tensionless string: high-energy limit $\alpha' \to \infty$ of string world-sheet

$$\alpha' \to \infty$$

$$\downarrow$$

window to tensionless (null) strings ?

<u>tensionless (null) string:</u> Carrollian limit $c \to 0$ on string world-sheet

ultra-relativistic limit

$$M_n := c\left(\mathcal{L}_n + \overline{\mathcal{L}}_{-n}\right) \qquad c_L := c_l - c_r \overset{cr.\ st.}{\to} 0$$

$$L_n := \mathcal{L}_n - \overline{\mathcal{L}}_{-n} \qquad c_M := c\left(c_l + c_r\right) \to 0$$

$$[L_n, L_m] := (n-m)\, L_{n+m} + \frac{c_L}{12}(n^3 - n)\, \delta_{n+m}\ ,$$

$$[L_n, M_m] := (n-m)\, M_{n+m} + \frac{c_M}{12}(n^3 - n)\, \delta_{n+m}\ , \quad [M_n, M_M] = 0$$

$CCS_2 \simeq CGal_2$

Bagchi, Chakrabortty, Parekh (2015)

- Carroll symmetry $CCS_2 \simeq BMS_3$ on closed string world-sheet

$$L_0 \,|\, \Delta, \xi\rangle = \Delta \,|\, \Delta, \xi\rangle$$

$$M_0 \,|\, \Delta, \xi\rangle = \xi \,|\, \Delta, \xi\rangle$$

Carrollian primary states

- similar for open (super)strings with boundary Carrollian symmetry

Bagchi, Chakrabortty, Chakrabortty, Fredenhagen, Grumiller, Pandit (2024)

Bagchi, Chakrabortty, Chakrabortty, Chatterjee, Pandit (2025) closed tensile string (entering our amplitude computations)

$$X^\mu(\sigma,\tau) = x^\mu + 2\sqrt{2\alpha'}\;\alpha_0^\mu\;\tau + i\sqrt{2\alpha'}\sum_{n\neq 0}\frac{1}{n}\left\{\;\alpha_n^\mu\;e^{-in(\tau+\sigma)} + \tilde{\alpha}_n^\mu\;e^{-in(\tau-\sigma)}\;\right\}$$

explicitly from closed tensile string

$$\begin{array}{l}\sigma \to \sigma \\ \tau \to c\tau\end{array} \quad \longrightarrow \quad \alpha' \to \frac{1}{c}\;,\; c \to 0$$

Bagchi, Chakrabortty, Parekh (2015)

ultra-relativistic limit

$$\updownarrow$$

$$X^\mu(\sigma,\tau) = x^\mu + \sqrt{2}\;A_0^\mu\;\sigma + \sqrt{2}\;B_0^\mu\tau + i\sqrt{2}\sum_{n\neq 0}\frac{1}{n}\left(A_n^\mu - in\tau\;B_n^\mu\right)e^{-in\sigma}$$

$$A_n^\mu = \frac{1}{\sqrt{c}}\left(\alpha_n^\mu - \tilde{\alpha}_{-n}^\mu\right)$$

$$B_n^\mu = \sqrt{c}\left(\alpha_n^\mu + \tilde{\alpha}_{-n}^\mu\right)$$

<u>Actually:</u>

$$\boxed{\begin{aligned} C_n^\mu &= \beta_+\;\alpha_n^\mu + \beta_-\;\tilde{\alpha}_{-n}^\mu \\ \tilde{C}_n^\mu &= \beta_-\;\alpha_{-n}^\mu + \beta_+\;\tilde{\alpha}_n^\mu \end{aligned}}$$

Bogoliuov transformations of oscillators

$$\beta_\pm = \frac{1}{2}\left(\sqrt{c} \pm \frac{1}{c}\right)$$

Only at $c = 1$
Strict disentanglement

- subleading corrections in amplitudes:

$$\frac{1}{\alpha'} \sim c$$

- double copy for tensionless string

double copy of open null string ?

(holomorphic factorization of world-sheet

likewise Carrollian CFT vs. boundary Carrollian CFT)

$$(h, \bar{h}) = \frac{1}{2}\left(\Delta + \frac{\xi}{c}, -\Delta + \frac{\xi}{c}\right) \stackrel{!}{=} (1,1)$$

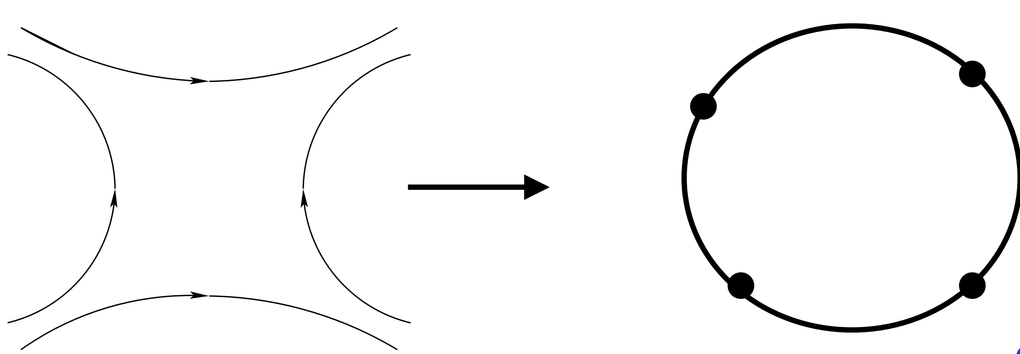
String World-Sheet and Celestial Sphere

$$\tilde{\mathcal{A}}_{\{\lambda_l\}}(\{z_l,\bar{z}_l\}) = \left(\prod_{l=1}^{n}\int_0^{\infty}\omega_l^{i\lambda_l}\,d\omega_l\right)\ \delta^{(4)}(\omega_1 q_1 + \omega_2 q_2 - \sum_{k=3}^{n}\omega_k q_k)\ \ A(\{\omega_i, z_i, \bar{z}_i\})$$

high-energy relation between
string world–sheet and celestial sphere

St.St., Taylor (2018)

<u>saddle point approximation</u> of the string amplitude
makes closest contact to the underlying <u>string world–sheet</u>



$\updownarrow$ $\quad \delta(z-\bar{z})$

$\alpha' = \infty \longleftrightarrow \sum_i \lambda_i = \infty$

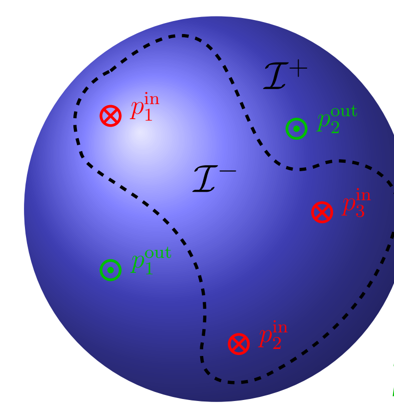


figure from Strominger

translates into <u>stationary phase expansion</u> of celestial amplitude
→ provides the most direct contact to the <u>celestial sphere</u>

string world-sheet *celestial sphere*

$$\sum_{i\neq j} \frac{p_i p_j}{z_i - z_j} = 0 \qquad z_{ij} = (\omega_i\omega_j)^{-1/2}\langle ij\rangle \longleftrightarrow \bar{z}_{ij} = (\omega_i\omega_j)^{-1/2}[ij] \qquad z_j = \frac{p_j^1 + ip_j^2}{p_j^0 + p_j^3} = \frac{p_j^0 - p_j^3}{p_j^1 - ip_j^2}\,, \quad j = 1,\ldots.n$$

saddle point equations (SEQ) *points on celestial sphere*

world-sheet vertex position = point on celestial sphere

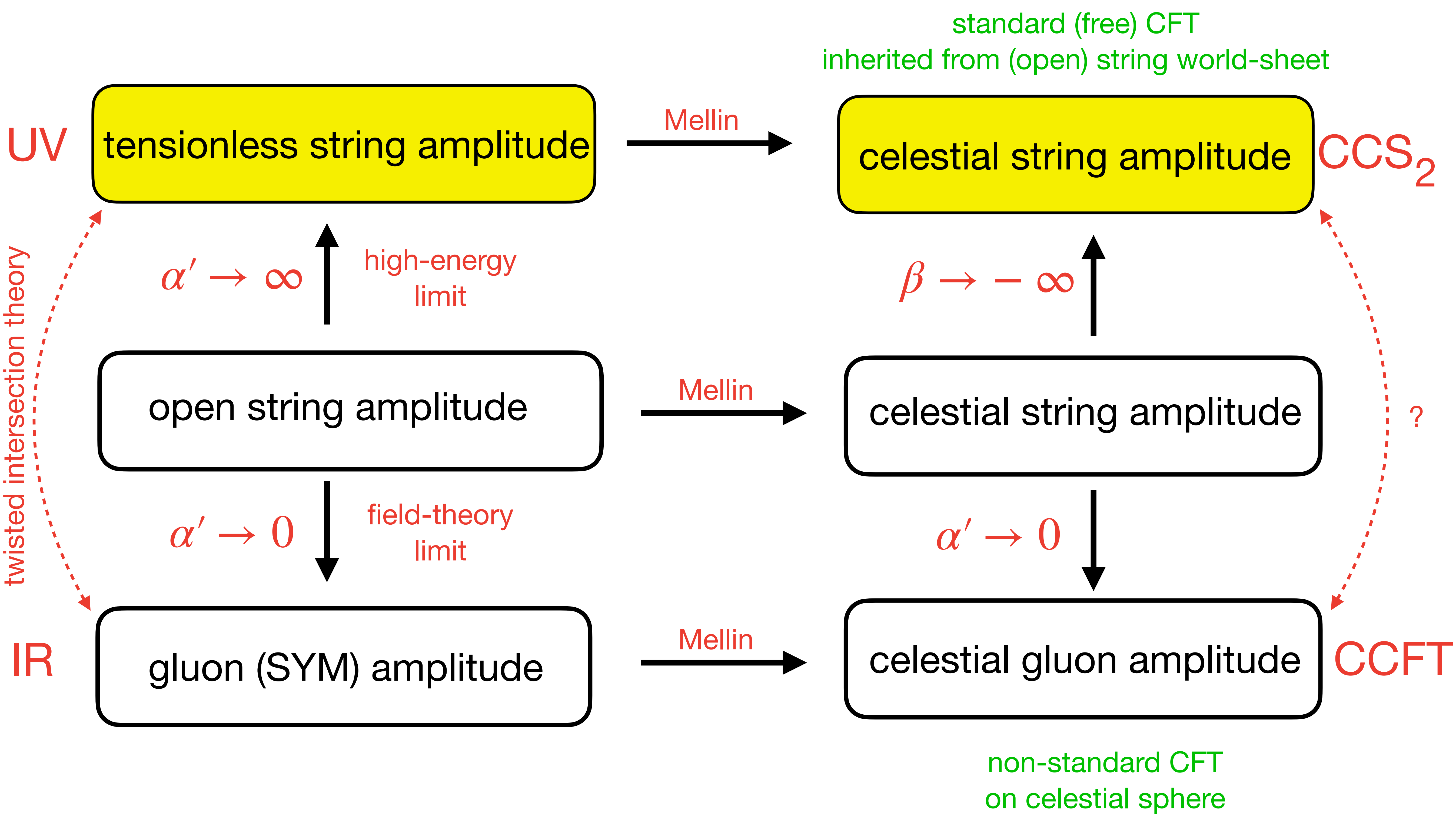
- may establish an underlying vertex operator description of CCFT

- may lead to an intrinsic construction of CCFT by relating it to a world–sheet CFT of tensionless string theory (CCS_2)

- *may lead to a rigorous definition of celestial geometry (top-down construction of CCFT)*

work in progress

Summary



standard (free) CFT
inherited from (open) string world-sheet
UV
tensionless string amplitude
Mellin
celestial string amplitude
CCS_2
$\alpha' \to \infty$
high-energy limit
$\beta \to -\infty$
twisted intersection theory
open string amplitude
Mellin
celestial string amplitude
?
$\alpha' \to 0$
field-theory limit
$\alpha' \to 0$
IR
gluon (SYM) amplitude
Mellin
celestial gluon amplitude
CCFT
non-standard CFT
on celestial sphere

- high-energy scattering in AdS: localize on the same saddle point

$$x_0 = \frac{1}{1-a} = -\frac{s}{t}$$

Alday, Hansen, Nocchi (2024)