Non-Lorentzian holography

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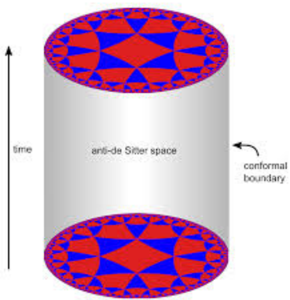
Introduction

- The most developed holographic dualities are those following from **AdS/CFT**, involving Lorentz invariant quantum field theories.
- However, dualities involving quantum theories without Lorentz symmetry have arisen many times in the last 25 years.
- **"Non-Lorentzian"** spans a wide range of holographic dualities, with a variety of interpretations and applications.
- Can we learn lessons from other dualities in developing **carrollian** and **celestial** holography?

Introduction: holographic dictionary

- Every **QFT operator** $\mathcal{O}_i$ and associated **source** ϕ_i can be mapped to bulk (gravity) data.
- We can compute **correlation functions** $\langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \cdots \rangle$ from the defining holographic relation for the partition function.
- QFT states are mapped to **curved geometries** (including thermal states/black holes).

AdS/CFT



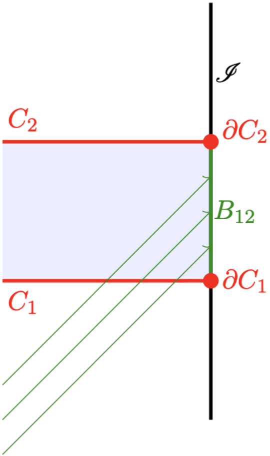
- The conformal boundary is timelike and describes the background geometry for the dual quantum field theory.
- Conformally flat boundary $\leftrightarrow$ Dual QFT on flat background is Lorentz invariant.
- Of course, a gravity/gauge theory duality for an asymptotically **locally** AdS spacetime will lead to a QFT on a **curved background**, non-Lorentz invariant.

Examples of QFT backgrounds

1. **Einstein Universe** $R_t \times S^D$:
   - Black hole properties; QFT in finite volume
2. **Riemannian** e.g. S^d, $S^p \times S^q$:
   - Localisation, exact results for SCFTs
3. Generic **asymptotically locally AdS**
   - Generic non-degenerate metric $g_{(0)}$ for dual QFT

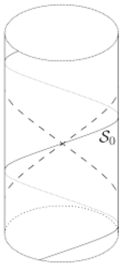
Holographic dictionary (renormalised onshell action, one point functions) does not rely on Lorentz symmetry of $g_{(0)}$.

Gravitational fluxes



- In AdS/CFT we can consider radiation reaching the conformal boundary in finite proper time.
- The boundary conformal structure is typically **time dependent** ($g_{(0)}(t)$), and charges are at most piece wise constant.
- Exact examples include **Robinson-Trautman**.

Degenerate timelike boundaries (AdS/CFT)



- Backgrounds with asymptotics

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2} g_{ij} dx^i dx^j + \frac{1}{z^{2\beta}} g_{ab} dy^a dy^b$$

  were considered early on in AdS/CFT.
- Analytic continuation of Kasner/BKL cosmological solutions.
- $\beta = 0$ is AdS $\times X$, with X capturing internal symmetries of dual CFT.
- Generic β: scale invariant, non-Lorentz invariant, dual theory.

AdS/CMT applications: Lifshitz

- The geometry

$$ds^2 = \frac{dz^2}{z^2} - \frac{1}{z^{2\beta_L}} dt^2 + \frac{1}{z^2} dx^i dx_i$$

  is clearly invariant under

$$z \to \lambda z; \qquad x^i \to \lambda x^i; \qquad t \to \lambda^{\beta_L} t$$

  i.e. **Lifshitz symmetry**, with Lorentz invariance broken for $\beta_L \neq 1$.
- We can engineer suitable fields (e.g. massive vector profile) to solve Einstein equations, although top down string constructions are sparse.

Holography for CFT versus Lifshitz

CFT:

- Given a (Lorentz invariant) CFT, we can straightforwardly couple to a curved background via $\eta \to g_{(0)}$.
- $g_{(0)ij}$ and $T^{ij} \sim \delta S / \delta g_{(0)ij}$ are conjugate, corresponding to non-normalizable/normalizable data for gravity.

Lifshitz:

- Curved backgrounds, coupled via vielbein, only studied in detail over last decade.
- Holographic dictionary complex, although principle of non-normalizable/normalizable data corresponding to curved background/stress-energy still applies.

Symmetry breaking and scaling limits

Starting from well understood AdS/CFT, we can access dualities with different symmetries:

- **Explicit and spontaneous symmetry breaking:**
  Best studied is RG flows, $SO(D+1,2) \to SO(D,1)$*, Lorentz preserving.*
- **Scaling limits:**
  Focussing on region of spacetime/subsector of quantum theory.

Deformations breaking Lorentz invariance

- One way to construct non-Lorentz invariant theories is by **deforming the CFT** with a suitable non-scalar operator e.g.

$$S_{\mathrm{CFT}} \to S_{\mathrm{CFT}} + \int d^d x \epsilon^\mu \mathcal{O}_\mu$$

- Particular choices: scale invariant, non-Lorentz invariant, theories e.g. **Schrödinger** and Lifshitz with $\beta_L \sim 1$.
- One can use conformal perturbation theory techniques, on both sides of the holographic dualiity.
- Typically holography is being used as a way to explore interesting classes of QFTs.

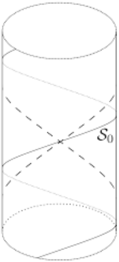
Scaling: Penrose limit, pp-wave and BMN

- **Penrose limit** of $AdS_5 \times S^5$ gives pp-wave

$$ds^2 = -2dudv - y^I y_I (dv)^2 + dy^I dy_I$$

  in which string theory is tractable (GS formalism).
- The limit corresponds to focussing on SYM operators with Δ and J (R charge) scaling as $\lambda = g_{YM}^2 N$, with $(\Delta - J)$ finite: **BMN matrix theory.**
- Holography: QFT correlation functions $\leftrightarrow$ string amplitudes but spacetime reconstruction (cf AdS/CFT) obscure.

Lessons from Lifshitz



- Many features of AdS/CFT retained: timeline conformal boundary; exemplar QFTs such as $\beta_L = 2$

$$S \sim \int dt d^2 x \left((\partial_t \phi)^2 - (\nabla^2 \phi)^2 \right)$$

- Yet asymptotically locally Lifshitz (boundary conformal structure $e^a_{(0)i}$) and holographic dictionary are subtle.

Lessons from scaling limits

$\Lambda \to 0$ limit of AdS/CFT gives **flat space/carrollian** theory.

- Leads to precise relations between **scattering amplitudes** and **correlation functions** etc.
- Some progress on geometric picture i.e. $\Lambda \to 0$ limit of **ALF spacetime reconstruction**.

but...

- Does this mean that carrollian/celestial theories should be viewed as EFTs, descending from parent CFTs?