De S-matrix in Deep IR

AdS/CFT meets Carrollian & Celestial Holography

ICMS, Edinburgh

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$$10^{44} = \frac{\text{Hubble} \leftarrow \text{IR}}{\text{LHC} \leftarrow \text{UV}}$$

or to say curvature > 0 but very small 10^{-44}

understand how **IR** decouple from **UV** (Appelquist Carrazzone)$^{-1}$

and then we want to dive deep **IR** $\sim 10^{-34}$ eV

so need to learn swimming first

$\longrightarrow$ S matrix

- role of observers & symmetries

0^{th} approx by flat Minkowski

Poincaré
10 generators

inertial observers
$\updownarrow$
observables

1^{st} approx by constant (small) curvature

compute observables — for whom?

→ maximal symmetry w/ 10 generators

ALL INERTIAL OBSERVERS EQUIVALENT

dS_4

$$-X_0^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 = \ell^2$$

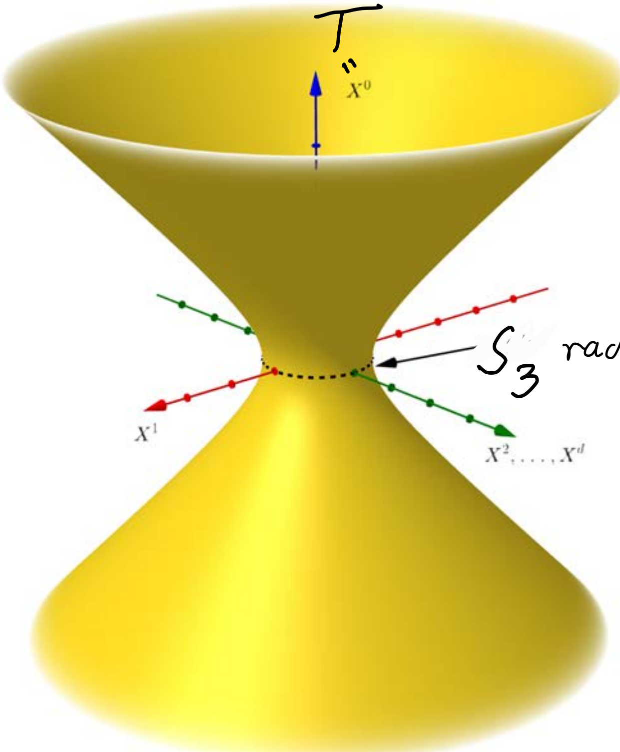
$$\downarrow$$

deS symmetry $SO(1,4)$ generated by L_{AB}

(Lorentz of M_5)

Mink_5

T



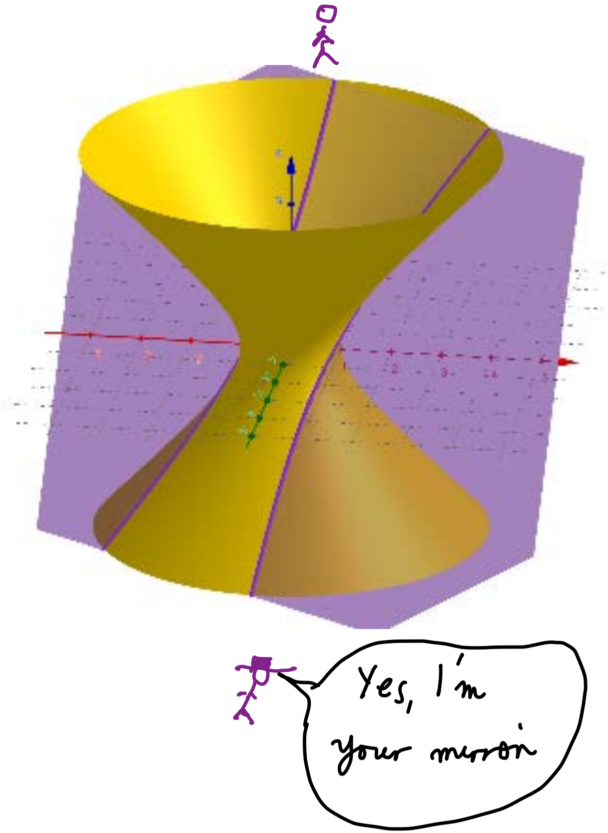
S_3 radius $\ell = 1$

$$deS \sim S_3 \times \mathbb{R}^1$$

$$ds^2 = \frac{-dt^2 + d\Omega^2}{\cos^2 t} \qquad t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

observers ride on time-like geodesics

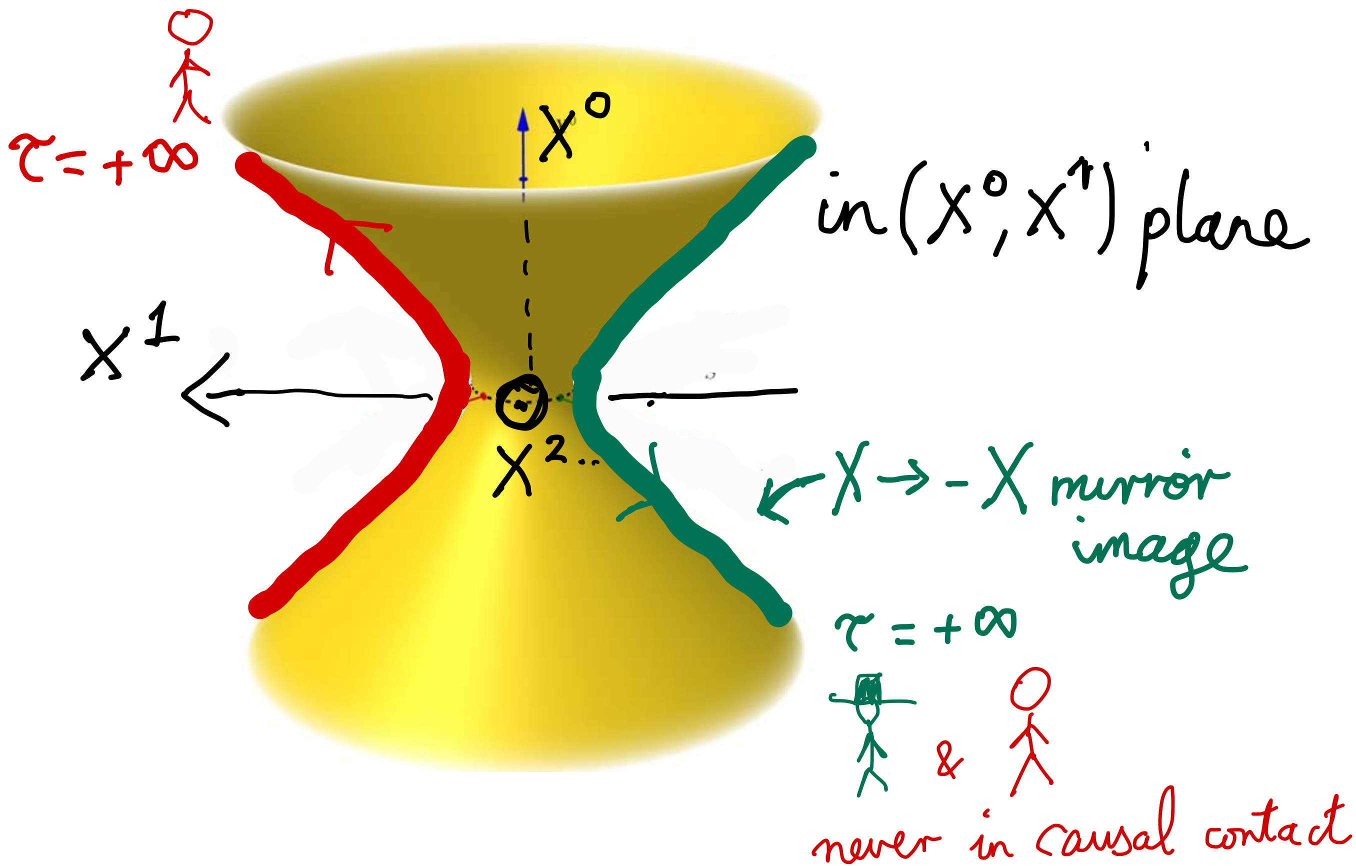
Observers



all time-like geodesics related by $SO(1,d)$

$\equiv$ inertial observers

Can be transformed into



Rule 1: $\quad dS_4 \xrightarrow[UV]{} M_4$

$$(\Box - m^2)\,\phi(t,\Omega) = 0$$

$\Box \sim \Delta_{S_3}$ $\qquad$ $\Omega \leftarrow S_3$

$$\phi(t,\Omega) = \sum_{\vec{L}} f_L(t)\, Y_{\underbrace{L}_{SO(4)}\underbrace{\ell\, m}_{SO(3)\ SO(2)}}(\Omega)$$

$\vec{L}$: $m = -\ell \ldots +\ell$

$\ell = 0, 1 \ldots L$

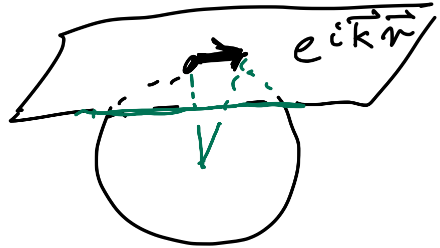
$L = 0, 1 \ldots \ldots$

$$\Delta_{S_3} Y_{\vec{L}} = -L(L+2)\, Y_{\vec{L}}$$

UV: $\tilde{L} \to \infty$

reps of $SO(1,4)$. $\quad |\vec{L}, \Delta, \cancel{spin}\rangle$

Actually, near observer



$L \sim |\vec{k}|$

$L \to \infty$, $|\vec{k}| \to \infty$

Particles : $f_L(t) \sim e^{-i\omega_L t}$ $\omega_L \sim L \to \infty$

$$f_L(t) \sim P^{\overbrace{-L-1}^{\text{order } m}}_{\underbrace{-\frac{1}{2} - i\mu}_{\text{degree } \ell}}(i\xleftarrow{M_5}T)$$

Wick-rotated $S_4 \to dS_4$

Chetyrkin Tagirov Mottola ...

$\mu = \sqrt{m^2 - 9/4}$

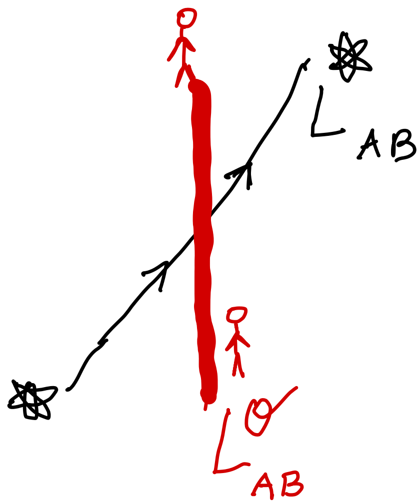
principal series $m^2 > \frac{9}{4}$

principal series: $\Delta = \frac{3}{2} \pm i\mu$

2. Unique deS vacuum

Bunch-Davies (Euclidean)

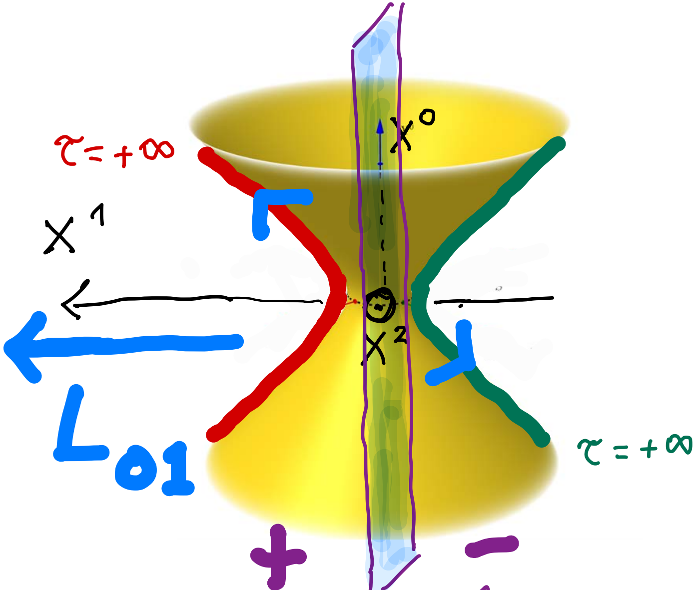
3. Hamiltonian : L_{01} boost generator



$\rightarrow \xi$ Killing vector

always time-like in observer's neighbourhood

$$E \sim \underbrace{L^{\mathcal{O}}_{AB}}_{\text{classical}} \; \underbrace{L^{AB}}_{\text{quantum}}$$ (Cacciatori et al)



$$H = L_{01} \leftrightarrow \text{Killing v. } \xi_\mu$$

"Energy current" $J^\mu = T^{\mu\nu}\xi_\nu$

$$H = \int J^0 \, d\Omega$$

$$H = \frac{1}{2}\int_0^{2\pi} \underset{d\chi\, d\theta}{d\varphi \sin\varphi} : \left[\left(\frac{\partial\phi}{\partial t}\right)^2 + \left(\frac{\partial\phi}{\partial\varphi}\right)^2 + m^2\phi^2 + H_I\right]:$$

4. S-MATRIX

$$ {}^{out}\langle \chi \,|\, \psi \rangle^{in} = \Big\langle \chi(0) \Big| \, e^{iH_0\tau} e^{-iH(\tau-\tau')} e^{-iH_0\tau'} \Big| \psi(0) \Big\rangle = \langle \chi(0) | \, U(\tau,\tau') \, | \psi(0) \rangle $$

Sidney Coleman

$$ U(\tau,\tau') = e^{iH_0\tau} e^{-iH(\tau-\tau')} e^{-iH_0\tau} $$

time evolution operator

$$ U(\tau \to \infty, \tau' \to -\infty) = T \exp\Big(-i \int_{-\infty}^{\infty} H_I(\tau)\, d\tau \Big) \,, $$

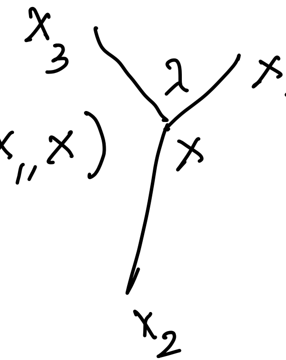
$$ {}^{out}\langle \chi \,|\, \psi \rangle^{in} = \Big\langle \chi(0) \Big| \, T \exp\Big(-i \int d^d x \sqrt{-g}\, H_I[\phi(x)] \Big) \Big| \psi(0) \Big\rangle $$

S-MATRIX Dyson's Formula

$|L, \ell, m, \Delta\rangle$

5. Feynman rules $dS_4 \longleftrightarrow M_4$ in position space

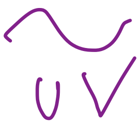
$$\sim i\lambda \int d^4x \sqrt{g}\, D_F(x_3, x)\, D_F(x_2, x)\, D_F(x_1, x)$$



Feynman Propagator in "Momentum" representation

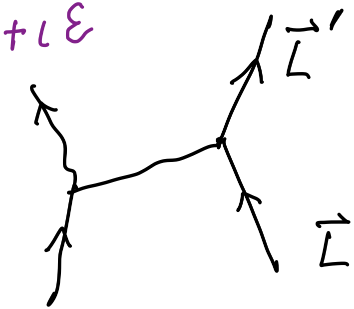
$$\int_0^\infty dt\,du\, F\left[t,u,\mu,\mathbf{L},T_1,T_2\right] \sum_L \int dk \, \frac{e^{ik(T_1-T_2)}}{-k^2+\underbrace{\left(\frac{t+u}{2}\right)^2}_{\text{virtual "packets"}}+i\varepsilon} \cdots \; Y_{\vec{\mathbf{L}}}(\Omega_1)\, Y^*_{\vec{\mathbf{L}}}(\Omega_2) \cdots$$

$$\downarrow$$



$$\int d^3\vec{L} \int dk \, \frac{e^{ik(T_1-T_2)}}{-k^2+\underbrace{\vec{L}^2+m^2}+i\varepsilon} \; e^{-i\vec{L}(\vec{x}_1-\vec{x}_2)}$$

saddle point integration

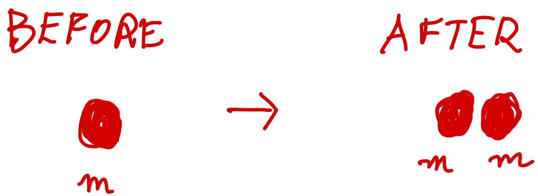


NO ON-SHELL POLES, NO IR SINGULARITIES

"BREIT-WIGNER PROPAGATORS"

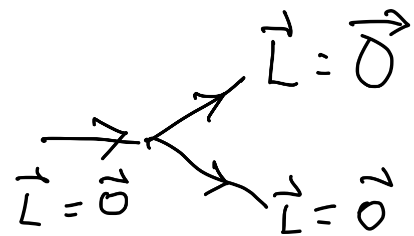
EXAMPLES OF DEEP IR

I. $1 \to 2$ decay, same mass:



AT REST: $\vec{L} = 0$

$$H_I = \frac{\lambda}{3!}\phi^3$$

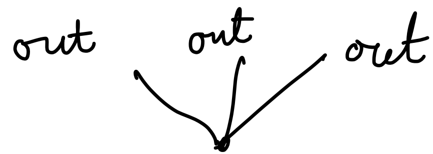


$${}_{out}\langle \vec{0}\,\vec{0} \,|\, \vec{0}\rangle^{in}\Big|_{\substack{\mu=0 \\ m=3/2}} \approx \frac{i\lambda}{\sqrt{2\pi}} * (0.52\ldots)$$

no stable particles

but $\sim e^{-m}$

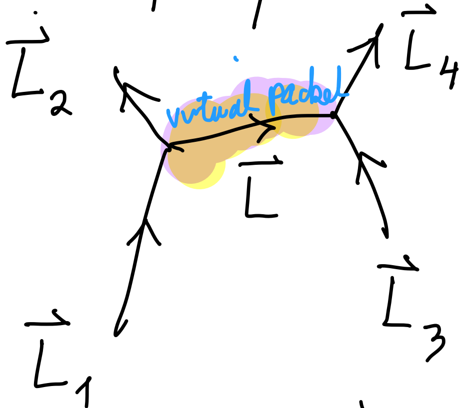
II Particle production from nothing



$$ {}^{out}\langle \vec{0}\ \vec{0}\ \vec{0}\ |\ \text{vacuum}\rangle^{in} = i\lambda \times \emptyset . $$

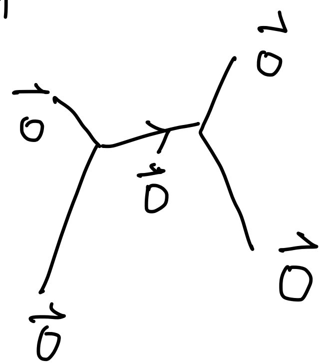
DeS is stable!

III 4-particle scattering



no "kinematic" singularities

like $\dfrac{1}{E^2-\vec{p}^{\,2}-m^2}$ poles



$$\sim (i\lambda)^2 \frac{1}{m^2-2} \xrightarrow{m^2 \gg 1} \frac{1}{m^2} \checkmark$$

$$\frac{1}{m^2-2} \xrightarrow{m^2=2} \infty$$

conformally coupled scalar
outside principal series

CONCLUSIONS

- we have De S-matrix
- 10 symmetry generators but algebra $\neq$ Poincaré
  $\rightarrow$ conservation laws different from Minkowski
- "Exotic" physics in deep IR $\sim 10^{-34}$ eV
  can we probe it ?
- IR cutoff $\rightarrow$ no kinematic singularities
  but IR divergences possible with conformal invariance
  WiP

THANK YOU!