K-multimagic squares and magic squares of kth powers via the circle method

Daniel Flores

Purdue University

Here we investigate K*-multimagic squares* of order N. These are $N \times N$ magic squares which remain magic after raising each element to the kth power for all $2 \le k \le K$. Given $K \ge 2$, we consider the problem of establishing the smallest integer $N_0(K)$ for which there exist *nontrivial* K-multimagic squares of order $N_0(K)$.

Previous results on multimagic squares show that $N_0(K) \le (4K-2)^K$ for large K. We use the Hardy-Littlewood circle method to improve this to

$$N_0(K) \le 2K(K+1)+1.$$

The intricate structure of the coefficient matrix poses significant technical challenges for the circle method. We overcome these obstacles by generalizing the class of Diophantine systems amenable to the circle method and demonstrating that the multimagic square system belongs to this class for all $N \ge 4$. We additionally establish the existence of infinitely many $N \times N$ magic squares of distinct kth powers as soon as

$$N > 2\min\{2^k,\lceil k(\log k +4.20032) \rceil \}.$$

This result marks progress toward resolving an open problem popularized by Martin Gardner in 1996, which asks whether a 3×3 magic square of distinct squares exists.