Counting commuting integer matrices

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Consider the set of pairs of d x d matrices A and B whose entries are all integers with absolute value at most N. We call (A, B) a commuting pair if AB = BA. Browning, Sawin, and Wang recently obtained an upper bound of order N^{d^2 + 2 - 2/(d +1)} for the number of commuting pairs. They further conjectured that the lower bound N^{d^2 + 1}, which comes from letting A or B be a multiple of the identity matrix, should be sharp (up to a factor depending only on d). In this talk, I will discuss recent joint work with Akshat Mudgal where we prove this conjecture for d = 2 and d = 3. I will also demonstrate how our approach relates counting commuting pairs of matrices to the study of restricted divisor correlations in number theory.