

Random Diophantine Equations with Prime Variables

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For many classes of Diophantine equations, one expects that if there are solutions in the reals and in every p -adic field, then there is a solution in the rationals. In particular, this is expected to hold for almost all homogeneous equations of degree $d \geq 3$ in at least $d+1$ variables (such an equation generically defines a Fano hypersurface). This was proven in all cases except the case of 4 variables in degree 3, by Browning, Le Boudec and Sawin. We discuss how to define similar criteria for when we expect an equation to have a solution in the primes, and how to adapt Browning, Le Boudec and Sawin's result to this case. We make use of the geometry of numbers and some bounds for counting points in lattices with prime coordinates.