Energy increment methods in Szemerédi-type theorems

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Let $[N] = \{1,2,\hdots,N\}$, and let $r_k(N)$ be the largest subset of $[N]$ with no k-term arithmetic progression. Szemerédi's theorem asks what bounds exist of $r_k(N)$ and concludes that $r_k(N) = o(N)$. There are many similar questions that can be asked, for example what if, instead of arithmetic progressions, one seeks a progression with differences defined by a polynomial. The question of ``popular" differences asks what can be said about the size of $N$ if one wants to guarantee the existence of a certain number of such progressions. In this talk, I will present a theorem of Peluse, Prendiville and Shao, answering one of these questions. I will focus in particular on their ``energy increment" step, outlining how such an argument can be used.