

ON THE THICKNESS AND GENERA OF CARTESIAN PRODUCTS OF GRAPHS

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(This talk is based on joint work with Elizabeth Badgett, Christian Millichap.)

The thickness and genera of graphs are well-studied invariants that suggest how far from the planarity of the graphs. In this talk, we discuss (i) the thickness of the Cartesian product of a complete graph and a path and (ii) a characterization of toroidal Cartesian products of graphs.

(i) We determine the thickness of the Cartesian product $K_{6p+4} \square P_2$ for $p \geq 0$ and of the Cartesian product $K_8 \square P_m$ for $m \geq 1$, where K_n and P_m denote the complete graph on n vertices and the path on m vertices, respectively. This almost solves the missing cases in the previous research.

(ii) A graph G is outer-cylindrical if there exists a planar embedding of G such that the vertices of G all lie on the boundary of at most two faces in this embedding. Let G and H be simple, connected, nontrivial graphs, and suppose G is 3-connected. Then $G \square H$ is toroidal if and only if G is an outer-cylindrical graph and $H = P_2$. This is the joint work mentioned above.