Rigidity of graphs on compact surfaces of genus at least two

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The theory of minimally rigid graphs in the Euclidean plane relies on matroidal and combinatorial methods, leading to a complete characterization of generic rigidity. Similar characterizations have been developed for frameworks on the sphere and the torus, but not for surfaces of higher genus. In this talk, I will describe a new approach to rigidity on compact surfaces of genus at least two using the hyperbolic geometry of their universal covers. By the Uniformization Theorem, every compact surface of genus at least two can be realized as a quotient of the hyperbolic plane by a surface group S. This allows frameworks on the surface to be viewed as infinite symmetric frameworks in the hyperbolic plane. Encoding the symmetry through group-labelled gain graphs, we show that infinitesimal rigidity can be determined entirely from finite combinatorial data. Our main result characterizes isostaticity in terms of gain sparsity: a hyperbolic framework is S-isostatic precisely when its associated gain graph satisfies the $(2, 3, 1, 0)$-gain-tight condition. The proof combines hyperbolic geometry, symmetric rigidity theory, and inductive constructions on gain graphs. As a consequence, we obtain a combinatorial characterization of generic rigidity for frameworks on compact surfaces of genus at least two.