Contraction Representation of Linear Delta-matroids with Applications

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(This talk is based on joint work with Tomohiro Koana.)

The classical representation of a linear delta-matroid $D = (V, \mathcal{F})$ is as a *twist* $D = D(A)\Delta S$, where $D(A)$ is the delta-matroid represented by a skew-symmetric matrix $A \in \mathbb{F}^{V \times V}$ over some field $\mathbb{F}$ and $S \subseteq V$ is the twisting set. However, this can be awkward for algorithmic applications. We consider an equivalent representation, as a *contraction representation* $D = D(A)/T$, where $A \in \mathbb{F}^{(V \cup T) \times (V \cup T)}$ is a skew-symmetric matrix over a larger ground set $V \cup T$. The two representations are equal in power, and in fact you can easily convert between them, but we find contraction representations much more convenient to work with. For instance, the following hold.

- The greedy algorithm (i.e. finding a max-weight feasible set) over D reduces to the greedy algorithm for a linear matroid.
- The union and delta-sum of linear delta-matroids, represented over the same field, are themselves linear.

The linear delta-sum operation in particular brings several applications, including:

- *Projected* linear delta-matroids $D = D'|X$ reduce to *elementary* projections, where $|X| = 1$.
- The Linear Delta-matroid Parity problem (and related problems such as Linear Delta-matroid Intersection) reduces to a linear independence check, in time $O(n^\omega)$, improving on $O(n^{\omega+1})$ by Geelen, Iwata and Murota. Here, $\omega < 2.373$ is the matrix multiplication exponent.
- The first polynomial-time (although randomized) algorithm for the Maximum Cardinality Intersection problem for linear delta-matroids.