Branch-width of represented matroids in matrix multiplication time

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(This talk is based on joint work with Tuukka Korhonen and Sang-il Oum.)

For an n-element matroid M given by an $n \times n$ matrix representation over a finite field $\mathbb{F}$ and an integer k, we present an $(O_{k,\mathbb{F}}(n^2) + O(n^\omega))$-time algorithm that either finds a branch-decomposition of M of width at most k, or confirms that the branch-width of M is more than k. Here, $\omega < 2.3714$ is the matrix multiplication exponent, and the $O_{k,\mathbb{F}}(\cdot)$-notation hides factors that depend on k and $\mathbb{F}$ in a computable manner. This improves the previous algorithms, including Hliněný and Oum [SIAM J. Comput. (2008)] and Jeong, Kim, and Oum [SIAM J. Discrete Math. (2021)], that run in at least $\Omega(n^3)$ time.

To suggest that our algorithm is optimal, we observe that for every field $\mathbb{F}$, deciding whether the branch-width of a matroid represented over $\mathbb{F}$ is 0 is as hard as deciding whether a square matrix over $\mathbb{F}$ is singular. Under the assumption that singularity testing requires $\Omega(n^\omega)$-time, this implies that the overhead of $O(n^\omega)$ is unavoidable. We also show strengthenings of this observation to rule out some approximations under this assumption.