

RECENT RESULTS ON LINEAR AND POLYNOMIAL χ -BOUNDEDNESS

Tung Nguyen

nguyent@maths.ox.ac.uk

University of Oxford, UK

A graph class $\mathcal{G}$ is χ -*bounded* if it is closed under taking induced subgraphs and there is a function $f: \mathbb{N} \rightarrow \mathbb{R}^+$ such that $\chi(G) \leq f(\omega(G))$ for all $G \in \mathcal{G}$. In this case, such a function f is called a χ -*binding function* for $\mathcal{G}$. The class $\mathcal{G}$ is *linearly* χ -*bounded* if it admits a linear χ -binding function, and *polynomially* χ -*bounded* if it admits a polynomial χ -binding function. We survey recent works on linear and polynomial χ -boundedness, and underline their connections to other topics such as Erdős–Hajnal and degree-boundedness. Results to be discussed include:

1. The class of graphs with no induced five-vertex path is polynomially χ -bounded.
2. For any path P and any complete multipartite graph M with parts of size two, the class of graphs with no induced P and no induced M is linearly χ -bounded.
3. For any tree T , the class of graphs with no induced T and no induced four-cycle is linearly χ -bounded.

(The last two results are joint with Sang-il Oum.)