Structure of non-disk faces of minimal surface embeddings

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A *closed-2-cell embedding* is an embedding of a 2-connected graph in a surface (torus, Klein bottle, etc) such that the closure of each face is homeomorphic to a closed disk. Call such faces *disk faces.* The *Strong Embedding Conjecture* states that every 2-connected graph has a closed-2-cell embedding in some surface. The Strong Embedding Conjecture is closely related to another well-known conjecture, the Cycle Double Cover Conjecture which states that every 2-connected graph contains a collection of cycles which covers each edge exactly twice. A closed-2-cell embedding of a graph provides a *cycle double cover* of that graph. These two conjectures are equivalent if the graph is cubic.

An embedding of a 2-connected graph G in a surface is a *minimal surface embedding* if G cannot be embedded in a surface with larger Euler characteristic (i.e., with the minimum Euler genus). If an embedding is not a closed-2-cell embedding, then some faces are not bounded by cycles of the graph, and consequently some facial walks contain repeated vertices, which means these faces touch themselves. Call such faces *non-disk faces.* A non-disk face in an arbitrary embedding can be very complicated.

In this talk we provide a structural result for non-disk faces of minimal surface embeddings.