k-R-ISIS, vSIS, BASIS and their Relation

Valerio Cini

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$$\text{given } g, g^{\alpha}, g^{\alpha^2}, \ldots, g^{\alpha^{k-1}}, \quad \text{compute } g^{\alpha^k}$$

Roadmap

- k-R-ISIS [ACLMT22]
- vSIS [CLM23]
- BASIS [WW23]

Short Integer Solution (SIS) [Ajtai96]

$$n\left\{ \overbrace{\mathbf{A}}^{m} \right. \cdot \mathbf{u} = \mathbf{0} \mod q$$

over $\mathbb{Z}_q$

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Example:

$$\mathcal{G} = \{1, X, X^2, \ldots, X^{k-1}\} \qquad \text{and} \qquad g^* = X^k$$

k-R-ISIS [ACLMT22]

Public parameters: $\mathbf{A}$, $\mathbf{t}$, $\mathbf{v}$ sampled uniformly at random

monomials $\mathcal{G} = \{g_1, \ldots, g_k\}$ challenge monomial g^*

k-R-ISIS [ACLMT22]

$$\mathbf{A} \cdot \underbrace{\begin{bmatrix} \mathbf{u}_1 & \cdots & \mathbf{u}_k \end{bmatrix}}_{\text{short preimages (the hints)}} = \mathbf{t} \cdot \overbrace{\begin{bmatrix} g_1(\mathbf{v}) & \cdots & g_k(\mathbf{v}) \end{bmatrix}}^{\text{monomials at public } \mathbf{v}} \bmod q$$

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Challenge: find short $(\mathbf{u}^*, s^*) \neq \mathbf{0}$ with $\mathbf{A}\,\mathbf{u}^* = \mathbf{t} \cdot g^*(\mathbf{v}) \cdot s^*$
for a challenge monomial g^* (s^* short slack)

k-R-ISIS [ACLMT22]

challenge monomial g^* + short slack s^*

$$\mathbf{A} \cdot \underbrace{\mathbf{u}^*}_{\text{the forgery (short)}} = \mathbf{t} \cdot g^*(\mathbf{v}) \cdot s^* \mod q$$

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vSIS [CLM23]

Public parameters: public random $\mathbf{v}$ (uniform)

monomials $\mathcal{G} = \{g_1, \ldots, g_\ell\}$

vSIS [CLM23]

$$\underbrace{\begin{bmatrix} \boxed{g_1(\mathbf{v})} & \boxed{g_2(\mathbf{v})} & \cdots & \boxed{g_\ell(\mathbf{v})} \end{bmatrix}}_{}^{g_i(\mathbf{v}):\ \text{monomials at public } \mathbf{v}} \cdot \boxed{\mathbf{p}} = \boxed{\mathbf{0}} \bmod q$$

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vSIS with $\mathbf{v} = v$ and $g_i(\mathbf{v}) = v^{i-1}$: $\mathbf{p}$ is a polynomial vanishing at v

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vSIS with $\mathbf{v} \in \mathbb{Z}_q^\ell$ and $g_i(\mathbf{v}) = \mathbf{v}_i$: standard SIS assumption

vSIS $\Rightarrow$ k-R-ISIS

use vSIS on $\{g_1, \ldots, g_\ell\}$ to solve k-R-ISIS: $\mathcal{G} = \{g_1, \ldots, g_{\ell-1}\}$, $g^* = g_\ell$

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$$\boxed{g_1(\mathbf{v}) \cdots g_{\ell-1}(\mathbf{v})} \cdot \boxed{\mathbf{r}_{\mathrm{t}}} = -\boxed{g_\ell(\mathbf{v})} \cdot \boxed{\mathbf{r}_{\mathrm{b}}}$$

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Notation 2/2

- $\mathbf{G} = \mathbf{I}_n \otimes \mathbf{g}$: *gadget* matrix, $\mathbf{g} = (2^0, 2^1, 2^2, \ldots, 2^{\lceil \log q \rceil - 1})$

$$\overbrace{\boxed{\quad \mathbf{G} \quad}}^{n\lceil \log q \rceil \text{ columns}} \cdot \mathbf{u} = \mathbf{t} \mod q$$

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every target $\mathbf{t}$ has a *short* preimage $\mathbf{u}$ under $\mathbf{G}$

(bit-decomposition: $\mathbf{u} = \mathbf{G}^{-1}(\mathbf{t})$)

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1. SIS is easy for $\mathbf{G}$, **2**. if $\mathbf{A} \cdot \mathbf{R} = \mathbf{G}$, then SIS is easy for $\mathbf{A}$.

BASIS [WW23]

$\mathbf{A}$ uniform random $\qquad \mathbf{A}_1, \cdots, \mathbf{A}_\ell, \mathbf{R} \leftarrow \mathsf{Samp}(1^\lambda, \mathbf{A})$

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Goal: solve SIS w.r.t. $\mathbf{A}$

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$\mathsf{BASIS}_{\mathsf{rand}}$ solution yields SIS solution for $\mathbf{A}_{i^*} = \mathbf{A}$

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$$\left[\begin{array}{ccc|c} \mathbf{A}_1 & & & -\mathbf{G} \\ & \mathbf{A}_2 & & -\mathbf{G} \\ & & \mathbf{A}_3 & -\mathbf{G} \end{array}\right] \overbrace{\begin{bmatrix} \mathbf{R}_1 & -\mathbf{R}_1 & \\ & & \\ & -\mathbf{R}_3 & \mathbf{R}_3 \\ & -\mathbf{I} & \end{bmatrix}}^{\mathbf{R}}$$

$$i^* = 2, \quad \mathbf{A}_1\mathbf{R}_1 = \mathbf{G}, \quad \mathbf{A}_3\mathbf{R}_3 = \mathbf{G}$$

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- Both hand out structured data tied to $\mathbf{A}_i = \mathbf{W}_i\mathbf{A}$
- BASIS$_{\text{struct}}$ trapdoor can heuristically be built from k-R-ISIS preimages (and viceversa)

Relation b/w k-R-ISIS and BASIS$_{\text{struct}}$

$$\boxed{\mathbf{A}} \cdot \boxed{\mathbf{u}_{i,j}} = \underbrace{\boxed{\mathbf{W}_i^{-1}} \cdot \boxed{\mathbf{y}_j}}_{g_{i,j}(\mathbf{V})} \mod q$$

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heuristically linearly independent

$\Downarrow$

(Ajtai) trapdoor for $\mathbf{B}_\ell$

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- Reductions for $\mathsf{BASIS}_{\mathsf{struct}}$ / k-R-ISIS / vSIS (or variants) in *useful* regimes (preliminary results for Decomposed-LWE)
- "Encryption friendly" vector/functional commitments from weaker assumptions?

k-R-ISIS

[C:ACLMT22]

k-R-ISIS

[C:ACLMT22]

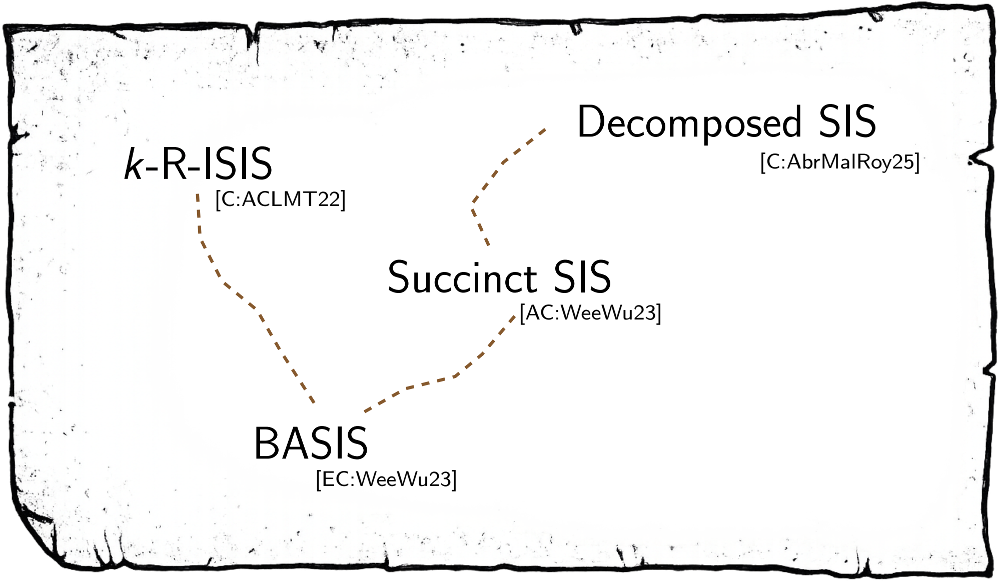
BASIS

[EC:WeeWu23]

k-R-ISIS
[C:ACLMT22]

Succinct SIS
[AC:WeeWu23]

BASIS
[EC:WeeWu23]


Decomposed SIS
[C:AbrMalRoy25]
k-R-ISIS
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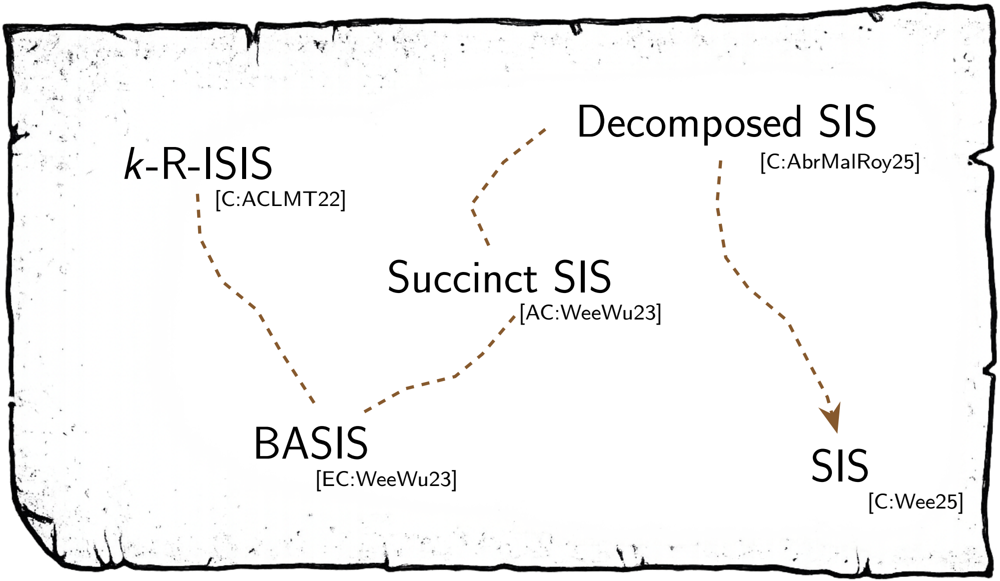
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Open Problems

- How does security of $\mathrm{BASIS}_{\mathrm{struct}}$ / k-R-ISIS / vSIS degrade with ℓ or k?
- Reductions for $\mathrm{BASIS}_{\mathrm{struct}}$ / k-R-ISIS / vSIS in *useful* regimes (preliminary results for Decomposed-LWE)
- "Encryption friendly" vector/functional commitments from weaker assumptions?

Thank you!