

On the Relation between Rectified Flows and Optimal Transport

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We investigate the connections between rectified flows, flow matching, and optimal transport. Flow matching is a recent approach to learning generative models by estimating velocity fields that guide transformations from a source to a target distribution. Rectified flow matching aims to straighten the learned transport paths, yielding more direct flows between distributions. Our first contribution is a set of invariance properties of rectified flows and explicit velocity fields. Our second contribution addresses recent claims suggesting that rectified flows, when constrained such that the learned velocity field is a gradient, can yield (asymptotically) solutions to optimal transport problems. We study the existence of solutions for this problem and demonstrate that they only relate to optimal transport under assumptions that are significantly stronger than those previously acknowledged. In particular, we present several counter-examples that invalidate earlier equivalence results in the literature, and we argue that enforcing a gradient constraint on rectified flows is, in general, not a reliable method for computing optimal transport maps.