Discrete Circles and their Intersections in Integer Geometry

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Integer geometry explores objects whose vertices lie on the integer lattice with congruence defined by lattice-preserving affine transformations. These objects are a possible discrete analogue for circles in Euclidean geometry, invariant in integer geometry, which challenge our geometric intuition. Unlike their classical counterparts, circles in integer geometry are unbounded, exhibit nontrivial arithmetic structure, and possess positive density in the plane. I will discuss pre-requisite notions in integer geometry for those unfamiliar with the area before introducing newer concepts of circles. The densities of integer circles and their intersections have expressions in the form of Euler products