Some Classification Questions: Algebraic Geometry vs. SQFT

Ron Donagi

Seiberg and Witten showed that some supersymmetric quantum field theories can be solved mathematically. Subsequent work suggested that every N=2 susy theory in 4D leads to an algebraically completely integrable system, which can be solved in terms of theta functions. The base of this system is the Coulomb branch of the theory, a vector space whose dimension is called the rank. The theory has a symmetry governed by the 'flavor' Lie algebra. A heroic series of works of Argyres et al gave a complete classification of the resulting integrable systems in rank 1. (Depending on exactly what you count, they get 35 or 28 or 27 such systems.)

In equally heroic works on the algebraic geometry side, Persson and Miranda classified all 279 types of rational elliptic surfaces in terms of their configurations of singular fibers. Subsequent work of Oguiso-Shioda determined the corresponding Mordell-Weil groups of sections, and Karayayla's thesis determined the automorphism groups.

In this talk we will explore the math/physics dictionary. The integrable systems arising from SQFT live on rational elliptic surfaces. Imposing some very simple constraints reduces the Persson-Miranda list to the physicists' list. Some natural expansions of this list arise from Karayayla's work. The flavor symmetry is interpreted in terms of the Mordell-Weil group. Many questions remain open.