

Heat kernel, Picard–Lefschetz and Alien

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We explain the resurgent structure of short-time heat kernel asymptotics from the viewpoint of Picard–Lefschetz theory. For a real analytic Riemannian manifold, the heat kernel admits a 1-Gevrey small-time expansion whose Borel transform detects complex-geometric data beyond the real geodesic sector. We formulate an infinite-dimensional Picard–Lefschetz problem of Morse–Floer type on the complexified path space, and propose a heat-kernel analogue of the Picard–Lefschetz/Alien correspondence. In this framework, pointed alien operators acting on the asymptotic expansion associated with the real geodesic are predicted to produce the formal heat-kernel sectors associated with other holomorphic geodesics, with coefficients given by signed counts of connecting trajectories of the Morse flow. Joint work with Xinxing Tang and Yong Li