

Let  $p$  be a fixed prime. A discrete  $p$ -toral group is a group that contains a normal subgroup  $T \cong (Z/p^\infty)^{\times r}$  of  $p$ -power index, where  $r \geq 0$ . A fusion system over a discrete  $p$ -toral group  $S$  is a category whose objects are the subgroup of  $S$  and whose morphisms are homomorphisms between them that satisfy a set of axioms. Fusion systems over discrete  $p$ -toral groups have been shown to admit a “classifying space” that is unique up to equivalence. Particular examples arise from finite groups (the finite case), compact Lie groups,  $p$ -compact groups and linear torsion groups. In all these cases the classifying space of the corresponding fusion system is homotopy equivalent to the  $p$ -completed classifying space of the object that gives rise to it. A fusion system over a finite  $p$ -group is said to be exotic if it does not arise from a genuine finite group. We found many such examples. Until recently it was not clear what should be meant by an exotic fusion system over an infinite discrete  $p$ -toral group. The project I will report on consists of two parts. In the first part we investigated four different ways of realising fusion systems over discrete  $p$ -toral groups by discrete groups, the most general of which we call “sequential realisability”. This means that the fusion system in question is in the appropriate sense a colimit of a sequence of  $p$ -local finite groups, each of which is realisable by a genuine finite group (but there need not be any nontrivial homomorphism between any of these groups). In the second part we concentrated on a specific very general type of discrete groups, and showed that for each such group one can associate a fusion system. A discrete group  $G$  is said to be 1) locally finite, if every finitely generated subgroup is finite, 2) a  $p$ -group if every element in it has a  $p$ -power order, and 3) artinian if it satisfies the descending chain condition. A group is said to be  $p$ -artinian if every  $p$ -subgroup of it is artinian. If  $G$  is locally finite and  $p$ -artinian, then every  $p$ -subgroup of  $G$  is locally finite and artinian, and hence is discrete  $p$ -toral by standard results. We show how to associate a fusion system to any locally finite  $p$ -artinian group  $G$  in such a way that the classifying space of the fusion system is homotopy equivalent to the  $p$ -completed classifying space of  $G$ . In doing so we need to deal with several nontrivial problems, one of which is that our groups need not have a Sylow  $p$ -subgroups in the usual sense. To prove the equivalence of the classifying spaces of the group and the fusion system we make use of a theorem of Gonzalez which expresses the cohomology of the classifying space of fusion system over a discrete  $p$ -toral group as the stable elements in the cohomology of the Sylow subgroup. In this talk I will start with the very essential Dave Benson connection. After introducing the basics I will concentrate on the second part of the project.