Numerical analysis of first order Mean Field Games under displacement monotonicity — Part 1

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In a recent joint work with Y. Osborne (Durham University), we introduced a particle method for the numerical approximation of time-dependent first-order Mean Field Games (MFGs) systems with non-separable, displacement monotone Hamiltonians and terminal costs, for arbitrary time-horizons and (possibly) singular initial player distributions. The numerical scheme is based on an implicit Euler discretisation in time and sampling in space of the characteristic Hamiltonian/Pontryagin system associated with the continuous MFGs system. We prove convergence of the approximations of the player distribution uniformly in time in the 2-Wasserstein metric and the approximations for the gradient of the value function along optimal trajectories uniformly in time in a suitable L^2-norm as the number of spatial samples tends to infinity jointly with the temporal time-step vanishing. The error bound that we establish for this convergence further implies rates of convergence of the scheme for a range of spatial sampling techniques. Provided that the Lagrangian and terminal costs are additionally locally Lipschitz continuous, we also establish an asymptotic error bound uniformly in time in the L^1-norm for the approximations of the value function along optimal trajectories. We illustrate the performance of the scheme in numerical experiments for a range of initial agent distributions, time horizons, and space dimensions. In this Part 1, we start first with the analytical framework for the continuous problem, at the heart of this work, while Part 2 will detail elements of the numerical analysis and simulations, and this will be presented in the talk of Y. Osborne.