

McKay's conjecture on character degrees: comments on the proof

Britta Späth

For any prime p , a finite group G has as many irreducible complex characters of degree prime to p as the normalizer of a Sylow p -subgroup. This equality, conjectured by John McKay in 1971, was reduced in 2007 by Isaacs–Malle–Navarro to a statement about linear and projective representations of finite simple groups.

In this talk I will discuss the recent proof of McKay's conjecture (Cabanès–Späth, 2026), which relies on the classification of finite simple groups. The main difficulties arise for simple groups of Lie type, which often combine non-trivial Schur multipliers with large (non-cyclic) outer automorphism groups. Among the methods used are the Jordan decomposition of characters (Lusztig), Shintani descent, and further tools developed specifically for this problem.

As by-products of independent interest we obtain, for any simple group G , a determination of $\text{Irr}(G)$ as an

$\text{Out}(G)$ -set and results on the extendibility of stable characters in many important normal inclusions. These developments have also contributed to the proofs of other global–local conjectures (Brauer height zero, Alperin–McKay for the prime 2 and play a role in ongoing work on further conjectures such as the Alperin weight conjecture and the Dade conjecture, which have already been reduced to questions about simple groups.