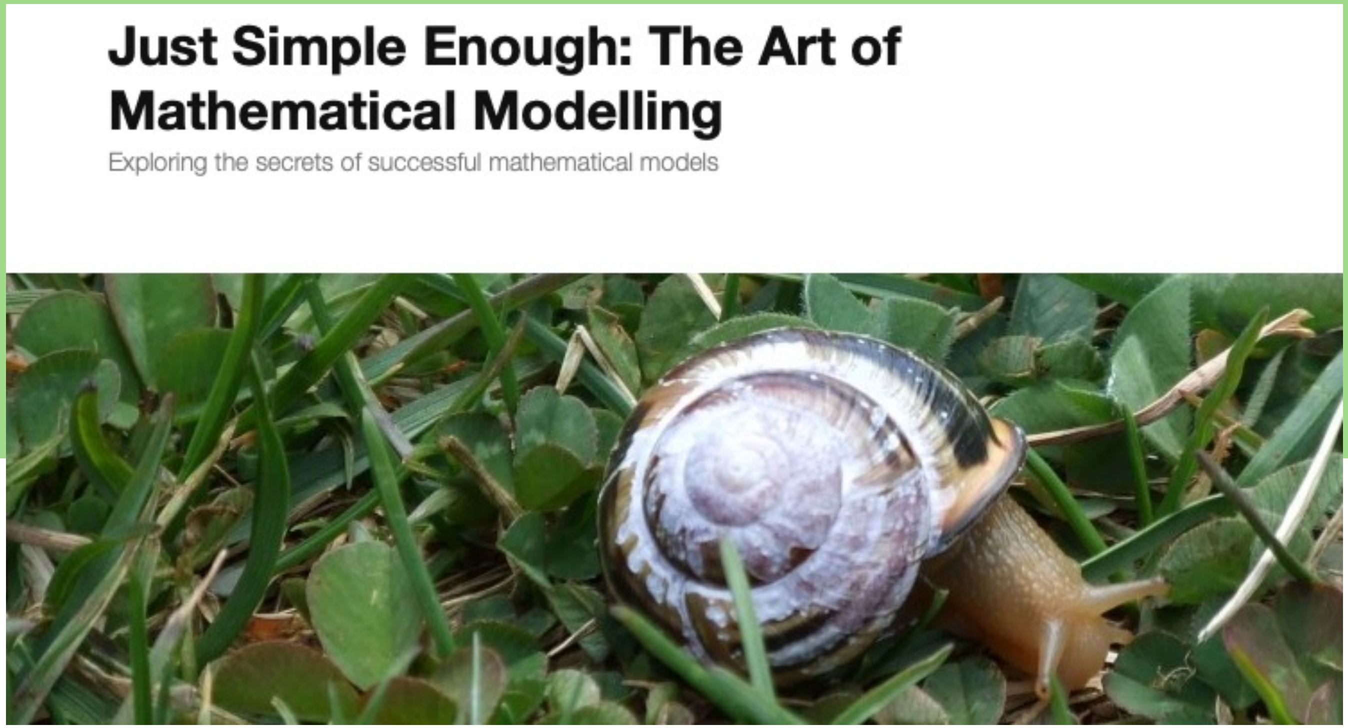
Modelling: a dialogue between nature and mathematics

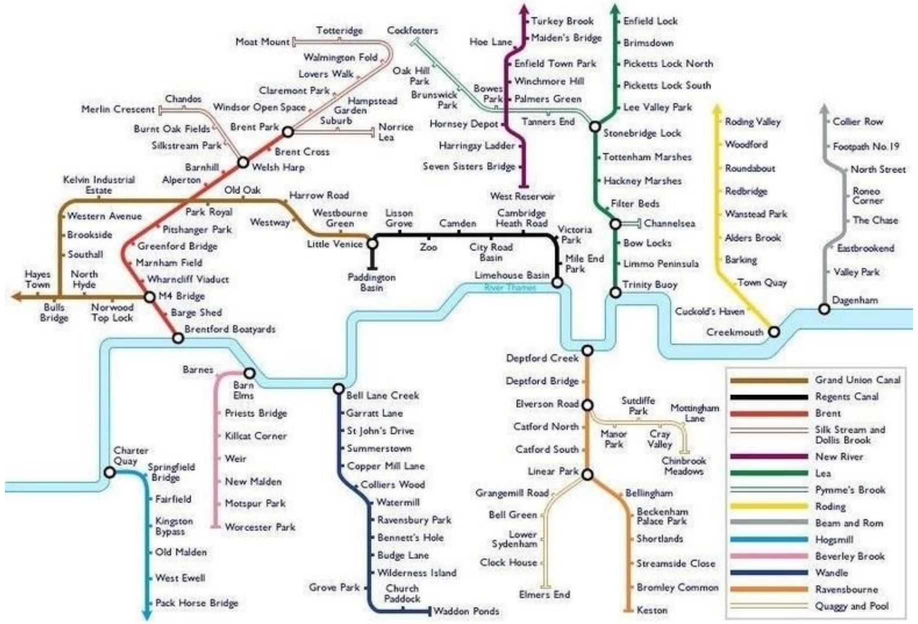


Christina Cobbold (and Amy Hurford!)

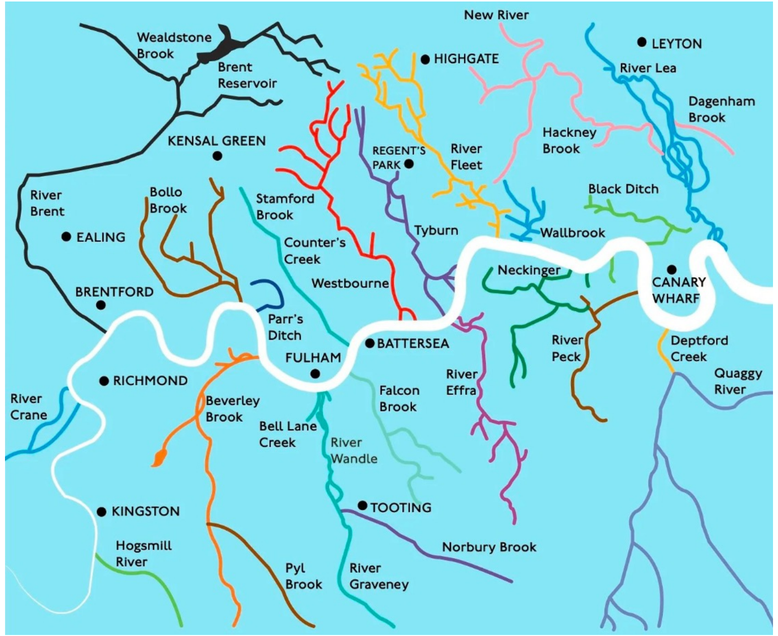
University of Glasgow

What constitutes a model?

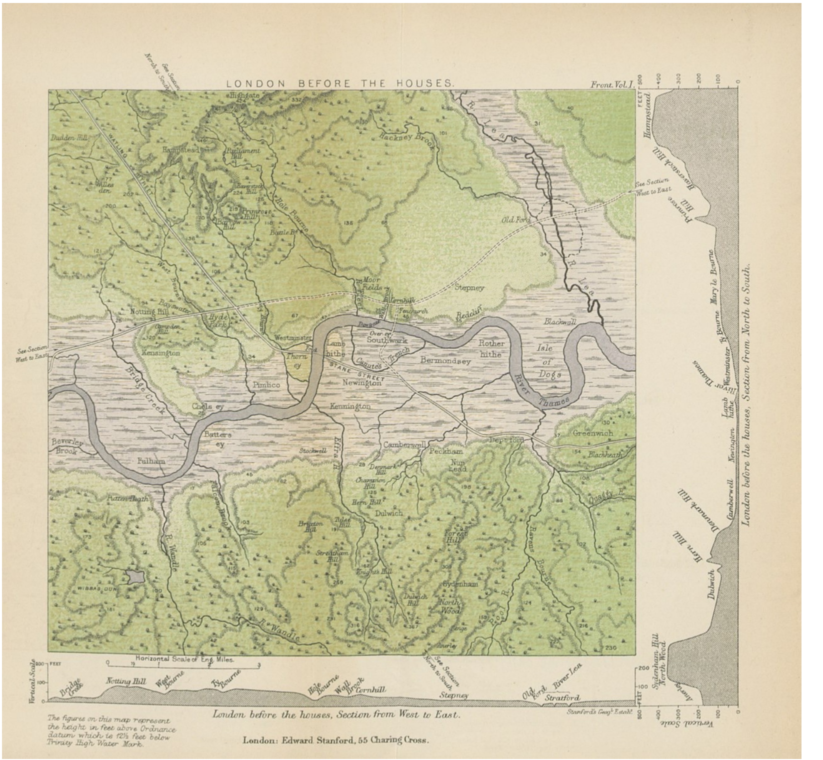
Model choice depends on the question to be answered



How can I navigate from A to B?



What is the distance from A to B?

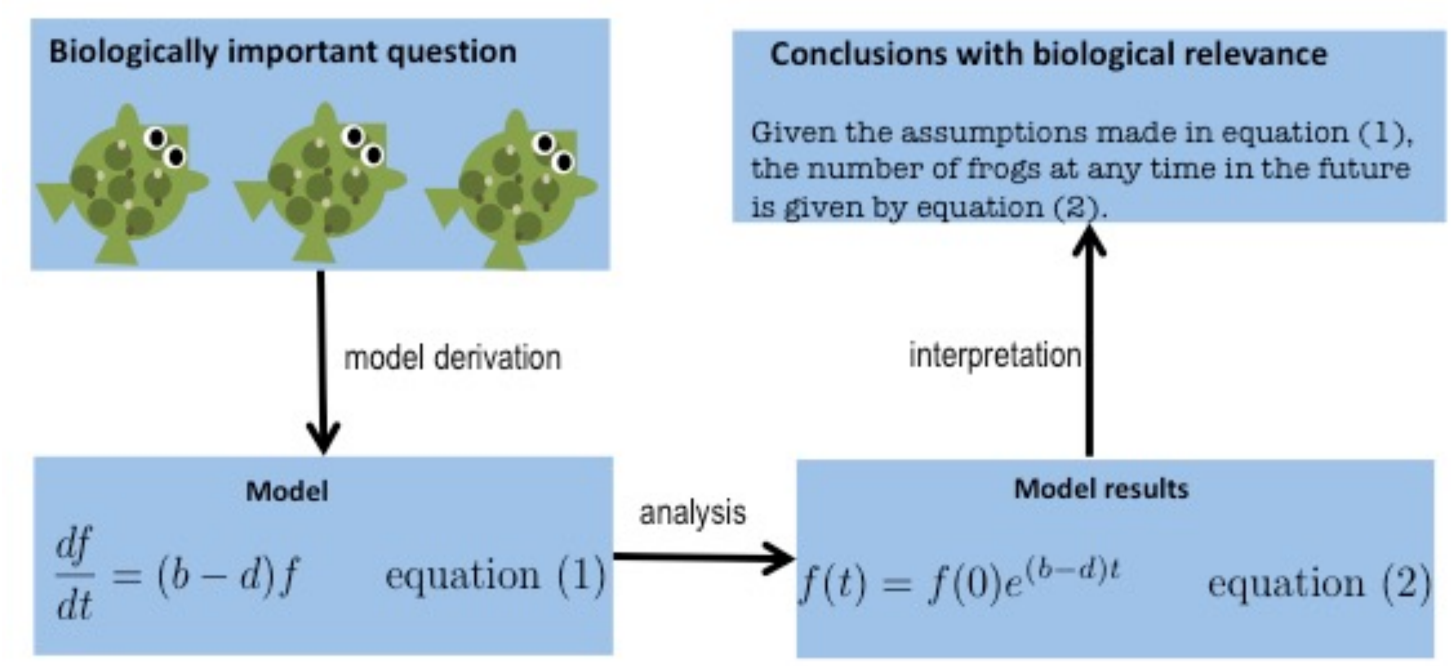


What is change in river height between A and B?

Building more complex models requires more knowledge or assumptions.

Mechanistic models

- A mathematical model is a set of mathematical constructs that represent the nature of biological, physical processes, or even processes that occur in other science or non-science disciplines (e.g. human behaviour and decision making)



Why make a mathematical model?

Our model should address a biological question. Some reasons:

1. **To make a qualitative (or quantitative) prediction**: Next year there will be fewer (or seven) frogs.
2. **To use information from one scale to understand another**: Given our knowledge of how individual frogs reproduce and disperse, we use the mathematical model to predict the future species range of the frog population.
3. **To investigate hypothetical scenarios** (and then make a quantitative or qualitative prediction). Particularly, scenarios on long time scales or large spatial scales for which it may be difficult to collect data or to conduct an experiment.
4. **To make an idea or hypothesis precise**, to integrate thinking and to think about a problem systematically.

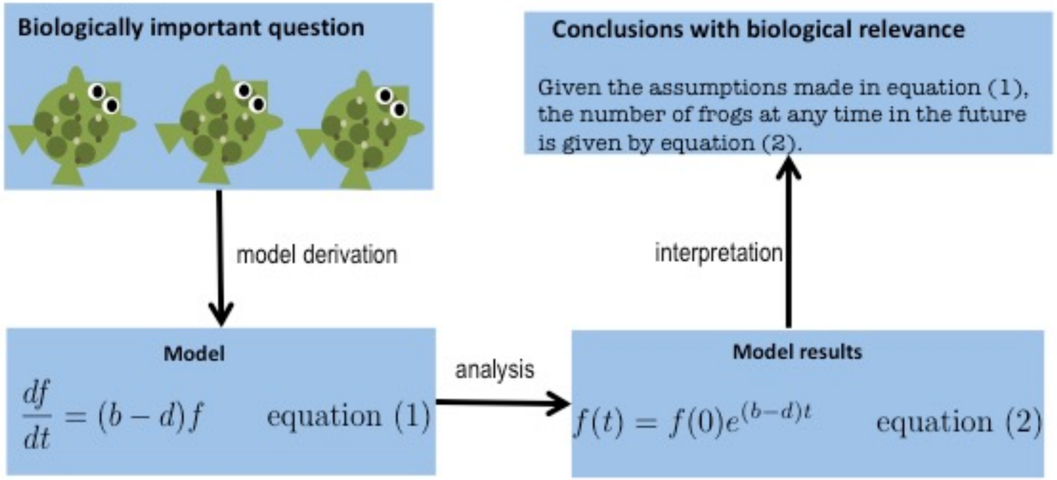
Why make a mathematical model?

Reasons contd:

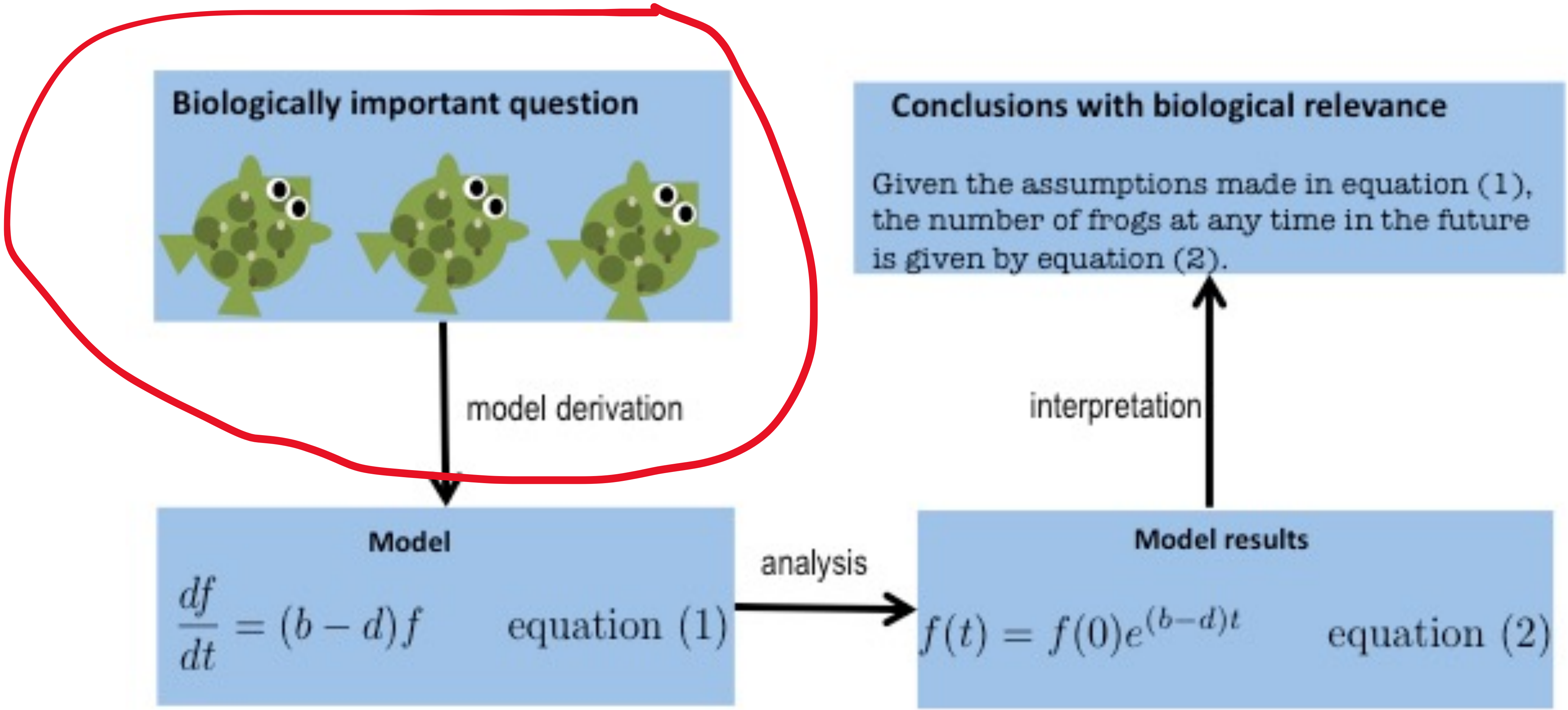
5. **To disentangle multiple causation**: Every year frogs die from either (a) old age, or (b) from Undetectable Frog Disease. The goal of the mathematical model is to show that time series of frog mortality data would exhibit tell tale signs of the relative contributions from (a) or (b), and determine what those signs are
6. To identify the processes or parameters that outcomes are most sensitive to
7. **To determine the necessary requirements for a given relationship**: Undetectable Frog Disease evolves intermediate virulence only under a convex trade-off between transmission and virulence.
8. **To identify common elements** from seemingly disparate situations.
9. To detect hidden patterns

Mechanistic models: what is the value of understanding?

"All other things being equal, mechanistic models tell you about the underlying processes driving patterns. They are more likely to work correctly when extrapolating beyond the observed conditions." Ben Bolker

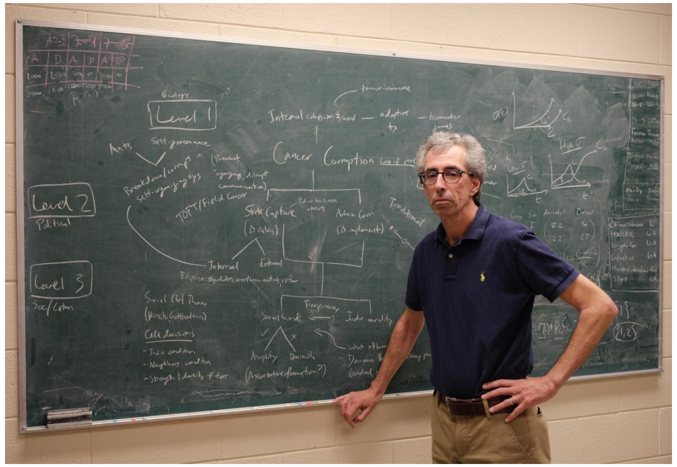


A dialogue between nature and mathematics



Building steps

- **Define the question the model should answer**
- **Identify the biological players involved**
  List the core components of the system. The selection of components should follow the principle "as simple as possible, as complex as necessary."



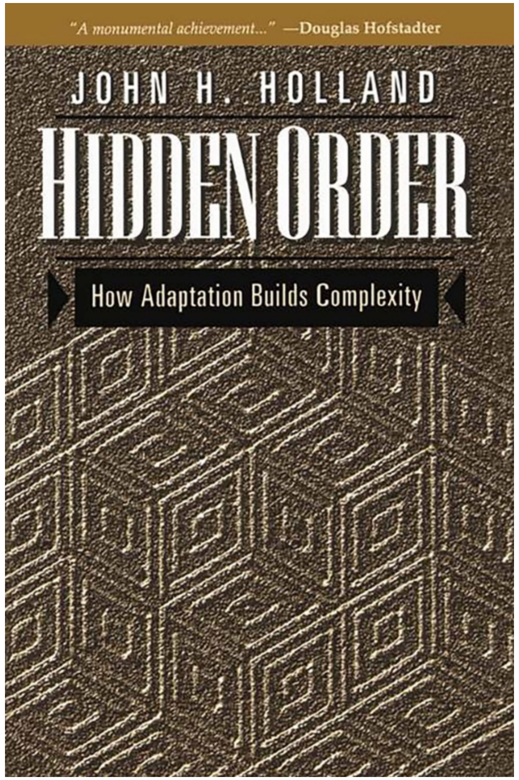
Fred Adler *lab group: "We neither fear complexity nor embrace it for its own sake, but rather face it with the faith that simplicity and understanding are within reach"*

Building steps

- **Identify the interactions and assign directionality**
  Make a diagram that represents your understanding of how the quantities you want to model change and interact

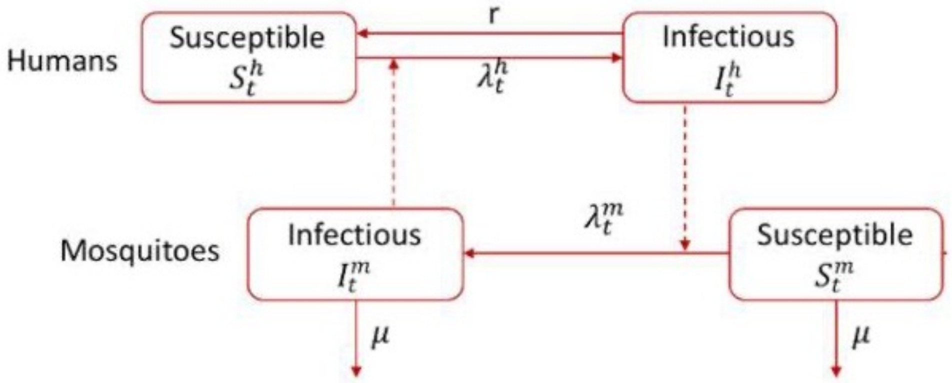
  A quote on parsimony

  "Model building is the art of selecting those aspects of a process that are relevant to the question being asked" – J.H Holland

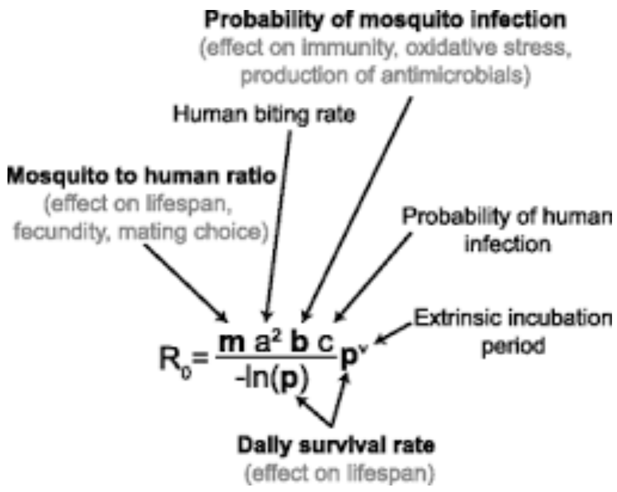


Example of a simple model: Ross' malaria model

- **Question:** Ronald Ross asked could malaria in north America be eradicated through the use of mosquito control?
- **Model**

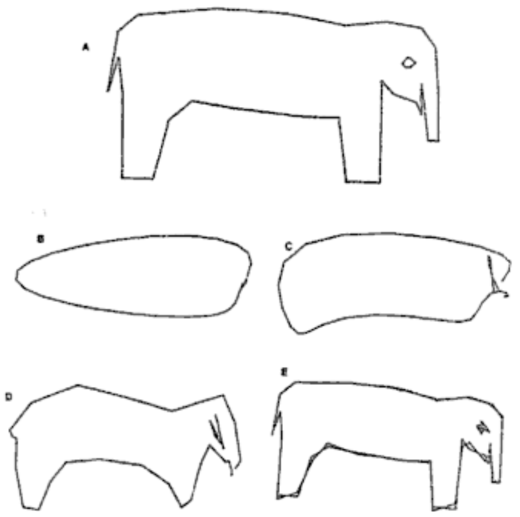


- **Model results:**



- Malaria could be eradicated as long as the number of mosquitoes per human was brought below a threshold value
- Malaria could then be eliminated without needing to kill every mosquito!

- **Simple models make a lot of assumptions**, but at the end of it you can conclude something concrete.
- **Complex models** still make assumptions, but they are a less restrictive type of assumption (i.e., **an assumption about how a factor is included rather than an assumption to ignore it**).
- The flexibility in complex models means that many different parameter combinations can lead to the same outcome



John von Neumann: *"with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."*

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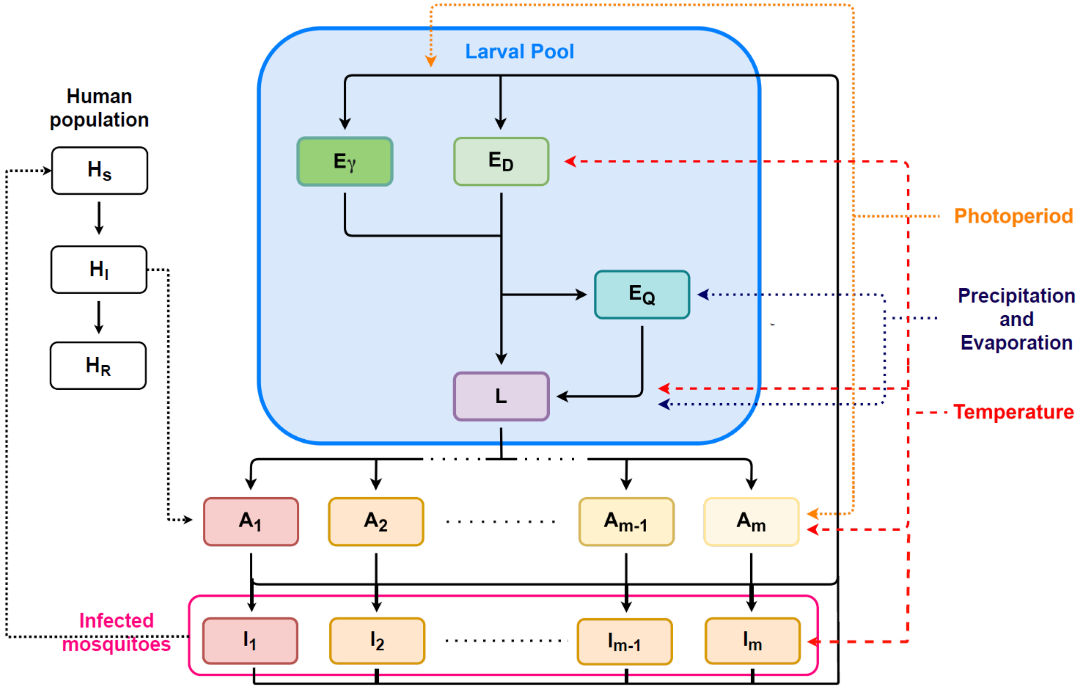
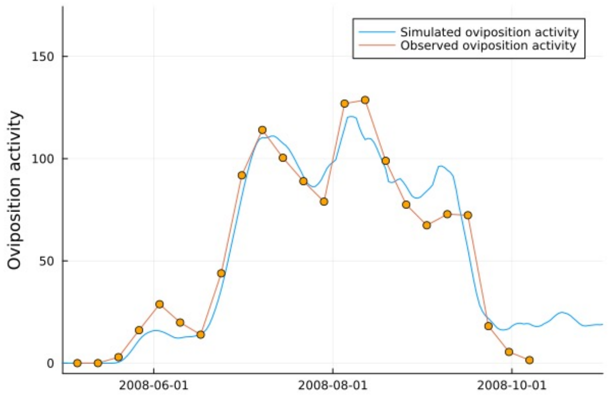
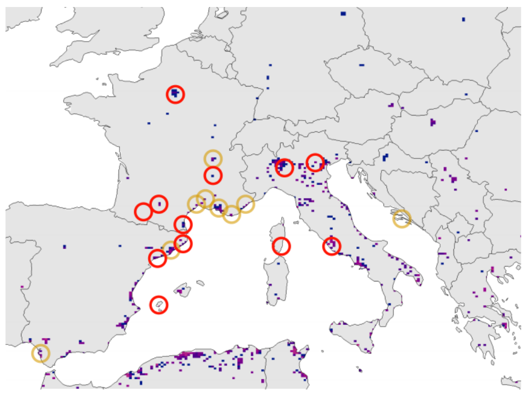
Example of a complex model: Brass et al model of dengue transmission

- **Question:** The mosquito vector can be found in many countries across the globe, but why do some regions get regular outbreaks and others do not?
- **Model** **Results**







Conclusions

- After I give a talk, I am often asked questions such as:
  - *“you assumed that space is homogeneous, but aren’t spatial dynamics important?”*

  or

  - *“couldn’t you expand your model to consider the influence of humidity, we know that’s really important?”*

  And for a split second this thought races through my head: *They’re right. I am wrong. My work is wrong! This is terrible, I must fix it.*
- Perhaps the questions to be asking are…..
  - What do I think would change if I had explicitly included this?
  - Is it important for the questions I am tackling?
  - Do I have enough information to include it in the model?