

Rational and Generic Cohomology: The Next Generation

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In honor of David Benson's 70th birthday, this talk explores the cohomology of finite groups of Lie type and its interactions with the representation theory of reductive algebraic groups.

A cornerstone of this subject is the 1977 result by Cline, Parshall, Scott, and van der Kallen (CPSvdK), which established a fundamental link between the rational cohomology of a connected reductive algebraic group G and the finite group cohomology of $G(\mathbb{F}_q)$.

Specifically, for a finite-dimensional rational G -module V and a fixed $n \geq 0$, the restriction map

$$\begin{aligned} & \text{res}: H^n(G, V^{(s)}) \rightarrow \\ & H^n(G(\mathbb{F}_q), V^{(s)}) \cong \\ & H^n(G(\mathbb{F}_q), V) \end{aligned}$$

becomes an isomorphism for sufficiently large s and r . For over three decades, this paper stood as the singular reference for the subject.

However, the 2013 work of Bendel, Nakano, and Pillen (BNP) fundamentally transformed the field by introducing a functorial approach to generic G -cohomology and its stability with rational G -cohomology. This framework provides not only new vanishing results for G and B -cohomology, but also stability ranges that significantly improve upon the seminal CPSvdK bounds, as well as refined vanishing ranges for $G(\mathbb{F}_q)$ that generalize the work of Hiller.

Despite these advancements becoming the current state-of-the-art approach, the BNP results are frequently omitted in recent literature, which continues to cite only the original 1977 paper. This talk serves as a necessary correction to the record, emphasizing that the BNP framework is now the essential reference for establishing modern stability and vanishing results.