Kolmogorov entropy of numerical solutions for scalar conservation laws with convex flux

Fabio Camilli

Motivated by the information-theoretic perspective introduced by P. D. Lax, we study the quantitative compactness of numerical solutions to scalar conservation laws with uniformly convex flux through Kolmogorov entropy. We prove that, under suitable grid conditions, conservative and monotone finite-difference schemes satisfying a discrete one-sided Lipschitz condition preserve the optimal entropy scaling of the corresponding exact entropy solutions.

The result is sharp and matches the classical continuous estimates established by De Lellis and Golse, and later by Ancona, Glass, and Nguyen. The upper bound follows from the discrete one-sided Lipschitz structure, while the lower bound is derived through a uniform approximation argument based on a precursor class of bounded-variation functions.

These findings provide a rigorous validation, in Lax's sense, of the high-resolution nature of prototypical first-order numerical schemes.

Finally, we formulate the mechanism underlying the lower bound as a general transfer principle, discuss its implications for information recovery through numerical post-processing, and outline several directions for future research.