An narrow band method for non-causal discretizations of anisotropic eikonal equations, with adaptive width, sub- and super-solution construction, and quasi-linear complexity

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We describe a narrow band numerical method for anisotropic eikonal equations, and more generally Hamilton-Jacobi-Isaacs equations with non-convex Hamiltonians, discretized using standard schemes (semi-Lagrangian, upwind-differences, Lax-Friedrichs). The causality property is not needed, in contrast with the fast marching method, and the convergence analysis instead relies on a contraction property of the schemes when expressed in Kruzkov coordinates.
The width of the narrow band is set adaptively based on the difference between a sub-solution and a super-solution, which are simultaneously constructed numerically. We prove that the worst case complexity is quasi-linear w.r.t. the number of discretization points, and logarithmic w.r.t. the tolerance parameter in the L-infty norm. Numerical experiments illustrate our results.