Hölder regularity for first-order non-coercive Hamilton-Jacobi equations associated with controllable linear systems

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Viscosity solutions of first-order Hamilton-Jacobi equations are known to be Hölder continuous (despite lacking parabolic structure) provided that the Hamiltonian has superlinear growth with respect to the momentum variable (e.g. Cardaliaguet, ESAIM:COCV 2009). However, many first-order Hamilton-Jacobi equations do not satisfy this hypothesis: for example, any Hamilton-Jacobi-Bellman equation associated with an optimal control problem in which, in the controlled ODE, certain directions are forbidden to the controller. In particular, this occurs for a large class of linear control systems. It is well known however that, under the Kalman rank condition, such linear systems are nonetheless controllable, such that any two points in the state space can be connected in positive time by a controlled trajectory.

I will present recent work considering first-order Hamilton-Jacobi equations structured on such linear control systems. We show that, assuming both the Kalman rank condition and a partial superlinear growth of the Hamiltonian respecting the controlled directions, viscosity solutions of such equations are Hölder continuous. The Hölder exponent is anisotropic, reflecting an underlying geometry induced by the control structure. This is joint work with Alpár Mészáros (Durham).