

Solving First-Order Time-Dependent Mean-Field Games via Monotone Operator Theory

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We study first-order, local, time-dependent mean-field games on the torus with a monotone Hamiltonian, using a variational inequality principle developed by Ferreira, Gomes, et al. (2018, 2019, 2021). We prove two main results. First, we establish the existence of a solution to the variational inequality via monotone operator theory in Banach spaces. Second, we show that any such solution satisfying natural integrability conditions is a weak solution in the sense introduced by Cardaliaguet (2015) for separable mean-field games. Together, these results establish the variational inequality principle as a general framework for mean-field games with non-separable Hamiltonians, well beyond the scope of minimization-based methods, yet recovering the same solution concept that those methods yield in the separable case.