The structure of biset functors

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The biset category has as its objects finite categories (or, more traditionally, finite groups) and morphisms are formal linear combinations of bisets, also known as profunctors or distributors. Biset functors are linear functors from this category to abelian groups. They originally arose in contexts having to do with classifying spaces and cohomology and are closely related to Mackey functors. Examples include representation rings, the Burnside ring, Hochschild cohomology and cohomology of the nerve of the category (with suitable modification of the biset category) and one reason for doing all this is to create a setting that will provide insight into these examples. It turns out that the biset category is rigid symmetric monoidal, giving a symmetric monoidal structure on the biset functors. The biset functors in special circumstances are a highest weight category. In other special circumstances, in work with Andrew Snowden, they are not only locally Noetherian but are locally finite length despite being non-Noetherian in general. We will give an overview of such structural properties.